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Data Assimilation for the Atmosphere, Ocean and Climate: Some Recent Results

Michael Ghil

Ecole Normale Supérieure, Paris, and
University of California, Los Angeles

Joint work with

D. Kondrashov & Y. Shprits, UCLA; C.-J. Sun, CSIRO, Perth; A. Carrassi, IRM, Brussels; A. Trevisan, ISAC-CNR, Bologna; A. Groth, ENS; P. Dumas & S. Hallegatte, CIRED; and many others: please see http://www.atmos.ucla.edu/tcd/ and http://www.environnement.ens.fr/

Outline

- Data in meteorology and oceanography
  - *in situ* & remotely sensed
- Basic ideas, data types, & issues
  - how to combine data with models
  - transfer of information
    - between variables & regions
  - stability of the forecast–assimilation cycle
  - filters & smoothers
- Parameter estimation
  - model parameters
  - noise parameters – at & below grid scale
- Subgrid-scale parameterizations
  - deterministic (“classic”)
  - stochastic – “dynamics” & “physics”
- Novel areas of application
  - space physics
  - shock waves in solids
  - macroeconomics
- Concluding remarks

Main issues

- The solid earth stays put to be observed, the atmosphere, the oceans, & many other things, do not.
- Two types of information:

  - direct $\rightarrow$ observations, and

  - indirect $\rightarrow$ dynamics (from past observations);

  both have errors.
- Combine the two in (an) optimal way(s)
- Advanced data assimilation methods provide such ways:

  - sequential estimation $\rightarrow$ the Kalman filter(s), and

  - control theory $\rightarrow$ the adjoint method(s)

Main issues (continued)

- The two types of methods are essentially equivalent for simple linear systems (the duality principle)
- Their performance differs for large nonlinear systems in:
  - accuracy, and
  - computational efficiency
- Study optimal combination(s), as well as improvements over currently operational methods (OI, 4-D Var, PSAS, EnKF).

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Atmospheric data

Drifting buoys: P_s – 267

Cloud-drift: V – 2x2259

Aircraft: V – 2x1100

Ship & land surface: P_s, T_s, V_s – 4x3446

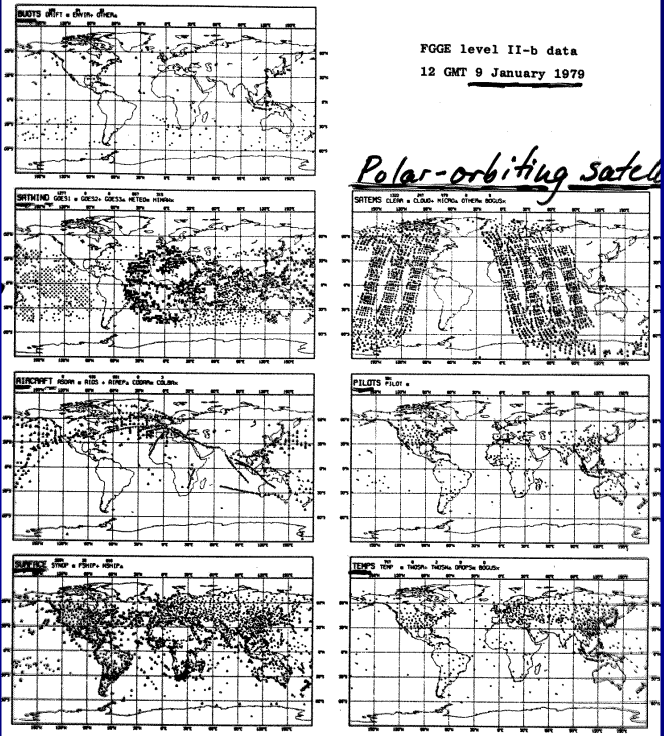


Fig. 1 An example of the different observing systems in use during the Global Weather Experiment.

Abbreviations used:

Aireps	Standard wind observations from aircraft
Asdars, Aids	High quality wind observations from aircraft
Buoys	Surface pressure observations from drifting buoys
Colba	Constant level balloons
Drops	Radiosondes dropped from aircraft
Pilots	Wind measurements from ascending balloons
Satems	Temperature measurements from polar orbiting satellites
Satwind	Cloud drift wind measurements from geostationary satellites
Ships	Surface observations from ships
Synops	Surface observations from land
Temps	Temperature, humidity and wind measurements from radiosondes

Polar orbiting satellites: T – 5x2048

Balloons : V – 2x581x10

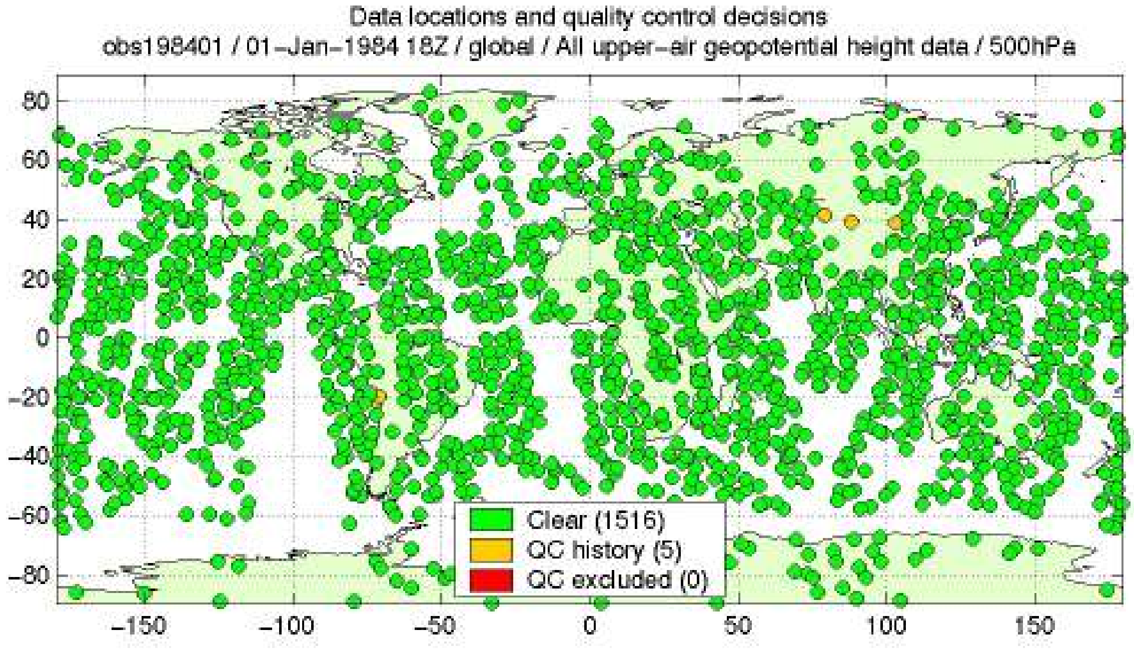
Radiosondes : T, V - 3x749x10

Total no. of observations = $0(10^5)$ scalars per 12h–24h

⁂ $0(10^2)$ observations/[(significant *d-o-f*) x (significant Δt)]

Bengtsson, Ghil & Källén (Eds.): *Dynamic Meteorology, Data Assimilation Methods* (1981)

Observational network



Quality control – preliminary & as part of the assimilation cycle

Ocean data – past

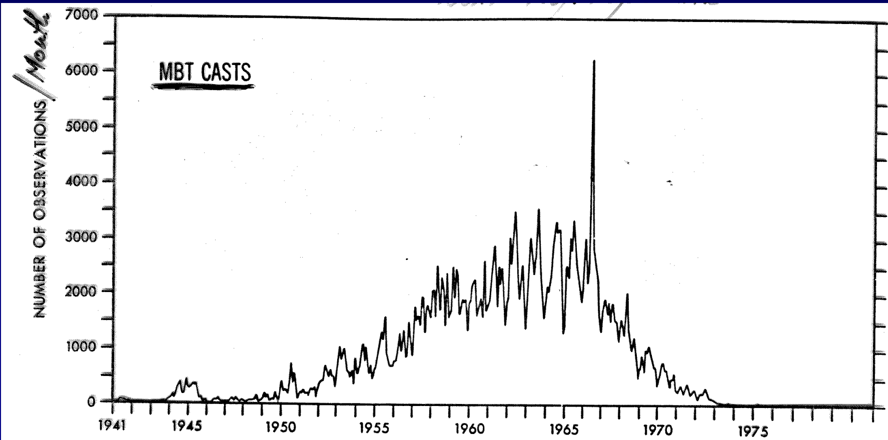


Figure 4.—Time series of MBT casts.

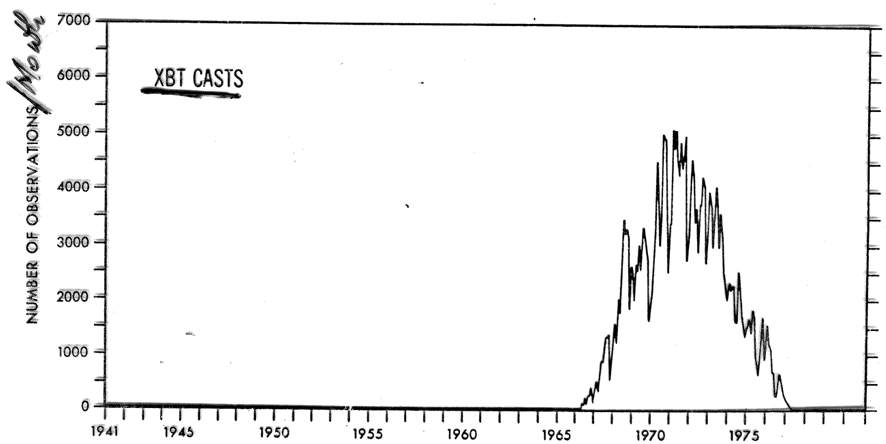


Figure 5.—Time series of XBT casts.

Total no. of (oceanographic observations)/(meteorological observations)

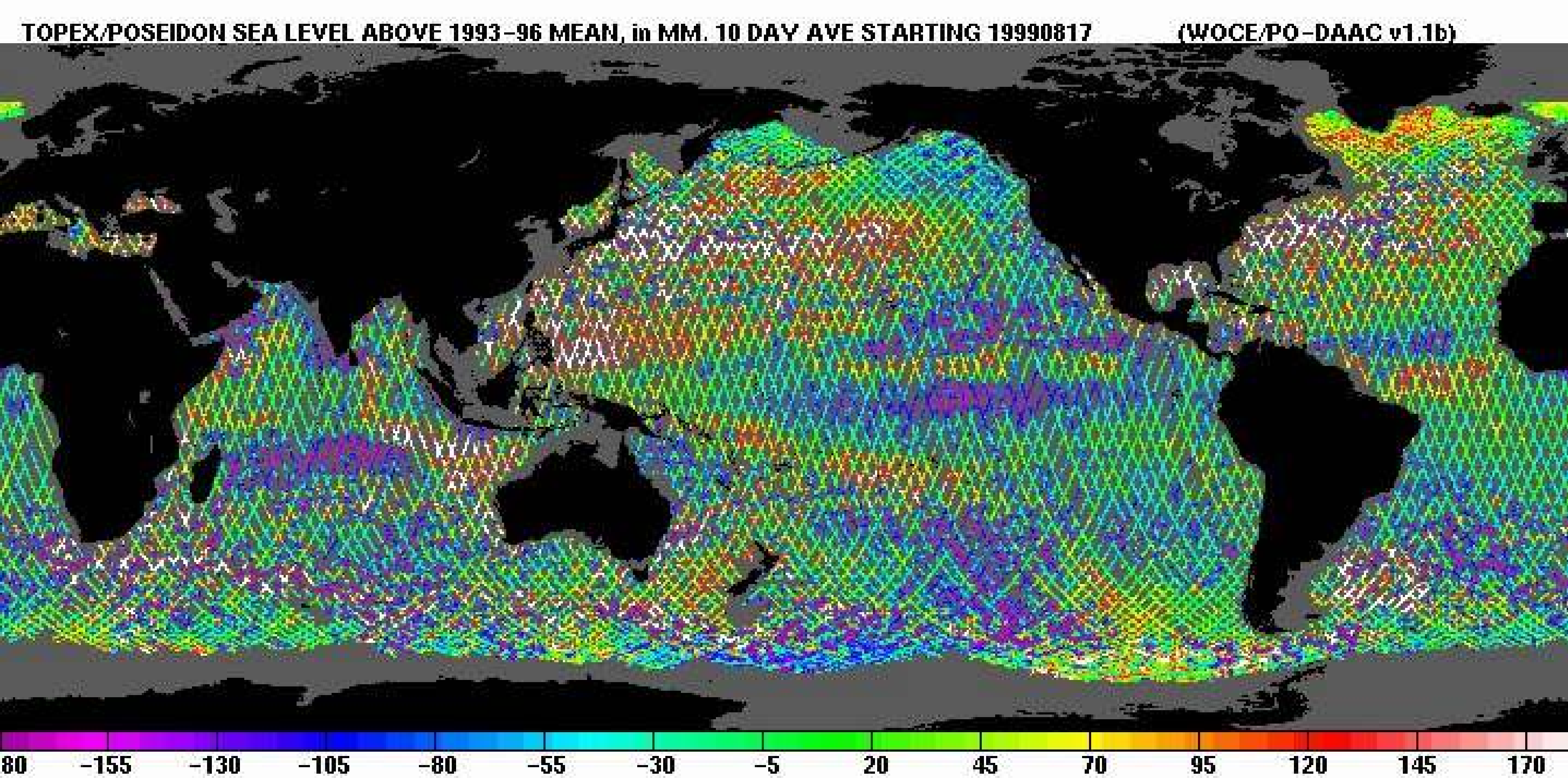
= $O(10^{-4})$ for the past; &

= $O(10^{-1})$ for the future :

Syd Levitus (1982).

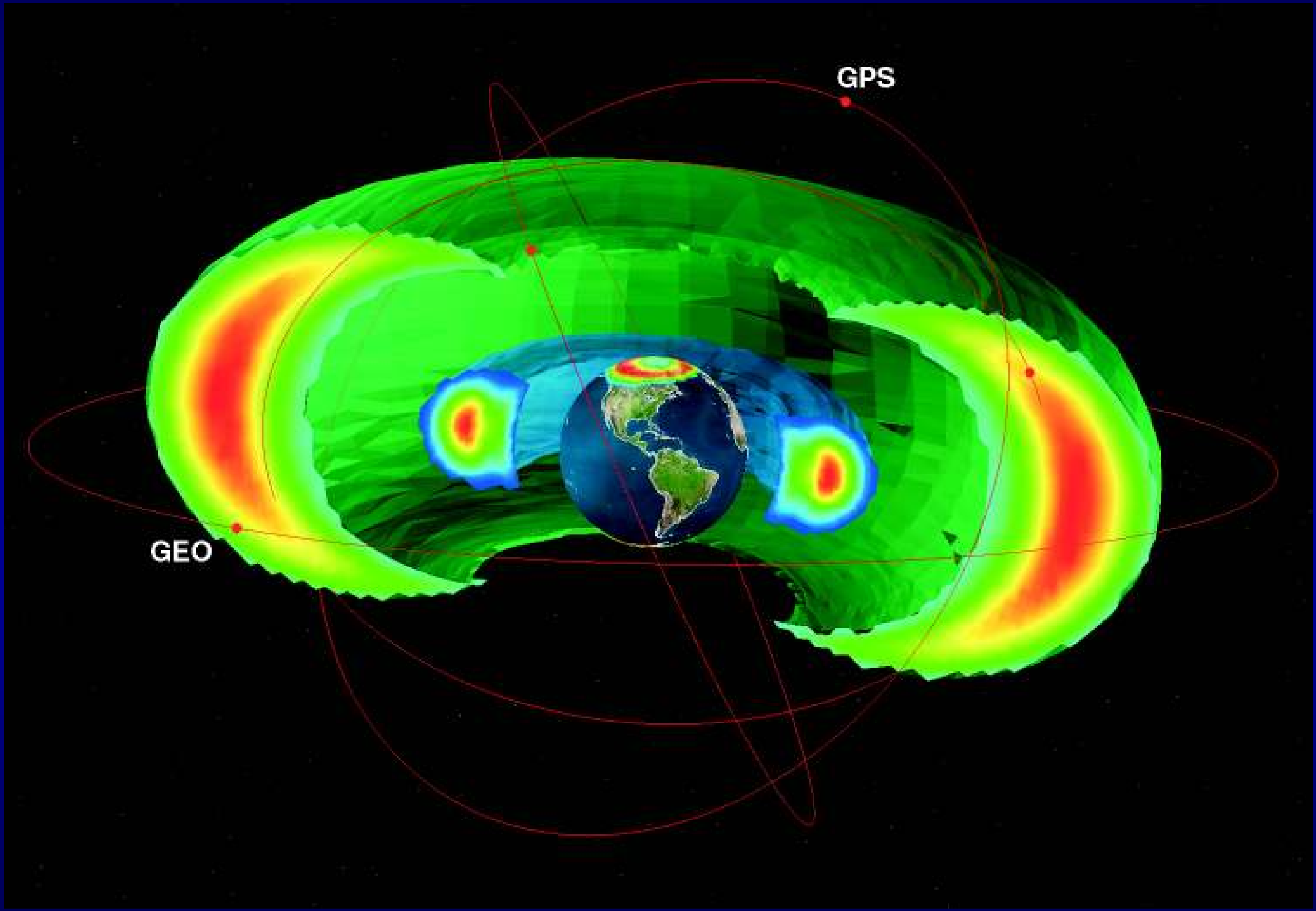
Ocean data – present & future

Altimetry ⇒ sea level; scatterometry ⇒ surface winds & sea state; acoustic tomography ⇒ temperature & density; etc.



Courtesy of Tong (“Tony”) Lee, JPL

Space physics data



Space platforms in Earth's magnetosphere

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Basic ideas of data assimilation and sequential estimation - I

Simple illustration

We want to estimate
T – the temperature of this room, based on the readings
T_1 and T_2 of two thermometers,

by a linear estimate $\check{T} = \alpha_1 T_1 + \alpha_2 T_2$.

The interpretation will be:
$T_1 = T_f$ - **first guess** (of numerical forecast model)
$T_2 = T_o$ - **observation** (R/S, satellite, etc.)
$\check{T} = T_a$ - **objective analysis**

Basic ideas of data assimilation and sequential estimation - II

If the observations T_1 and T_2 are unbiased, and we want $\check{T}$ to be unbiased, then

$$\alpha_1 + \alpha_2 = 1,$$

so one can write $\check{T} = T_1 + \alpha_2(T_2 - T_1)$: updating (sequential).

If T_1 and T_2 are uncorrelated, and have known standard deviations,

$$A_1 = \sigma_1^{-2},\ A_2 = \sigma_2^{-2},$$

then the minimum variance estimator(*) is

$$\check{T} = T_1 + [A_2 / (A_2 + A_1)]\,(T_2 - T_1)$$

and its accuracy is

$$\hat{A} = (A_1 + A_2) \geq \max\{A_1, A_2\}.$$

(*) BLUE = Best Linear Unbiased Estimator

(Extended) Kalman Filter (EKF)

True Evolution (deterministic + stochastic)

$$\mathbf{x}^t(t_{i+1}) = M_i[\mathbf{x}^t(t_i)] + \eta(t_i)$$

$$\mathbf{Q}_i\delta_{ij} \equiv \mathbb{E}(\eta_i\eta_j^T)$$

Observations

$$\mathbf{y}_i^0 = H_i[\mathbf{x}^t(t_i)] + \varepsilon_i$$

$$\mathbf{R}_i\delta_{ij} \equiv \mathbb{E}(\varepsilon_i\varepsilon_j^T)$$

$\mathbf{d} = \mathbf{y}_i^0 - H_i[\mathbf{x}^f(t_i)]$ - innovation vector

$$\Delta\mathbf{x}^{f,a} \equiv \mathbf{x}^{f,a} - \mathbf{x}^t$$

$$\mathbf{P}^{f,a} \equiv \mathbb{E}[(\Delta\mathbf{x}^{f,a})(\Delta\mathbf{x}^{f,a})^T]$$

$\mathrm{tr}\mathbf{P}^{f,a} =$ global error

Stage 1: Prediction (deterministic)

$$\mathbf{x}^f(t_i) = M_{i-1}[\mathbf{x}^a(t_{i-1})]$$

$$\mathbf{P}^f(t_i) = \mathbf{M}_{i-1}\mathbf{P}^a(t_{i-1})\mathbf{M}_{i-1}^T + \mathbf{Q}(t_{i-1})$$

Stage 2: Update (Probabilistic)

$$\mathbf{x}^a(t_i) = \mathbf{x}^f(t_i) + \mathbf{K}_i(\mathbf{y}_i^0 - H_i[\mathbf{x}^f(t_i)])$$

$$\mathbf{P}^a(t_i) = (\mathbf{I} - \mathbf{K}_i\mathbf{H}_i)\mathbf{P}^f(t_i)$$

$$\mathbf{K}_i = \mathbf{P}^f(t_i)\mathbf{H}_i^T[\mathbf{H}_i\mathbf{P}^f(t_i)\mathbf{H}_i^T + \mathbf{R}_i]^{-1}$$

subject to $\partial_{\mathbf{K}}\mathrm{tr}\mathbf{P}^a = 0$

$\mathbf{M}$ and $\mathbf{H}$ are the linearizations of M and H

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Basic concepts: barotropic model

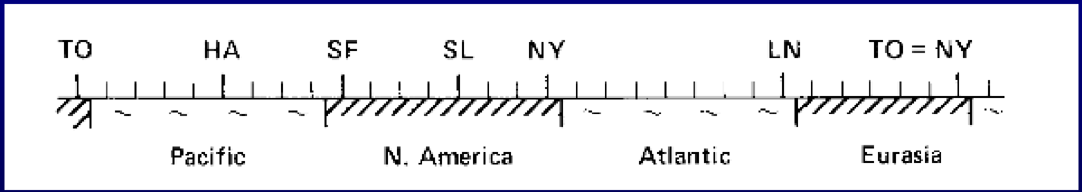
Shallow-water equations in 1-D, linearized about (U, 0, Φ), $fU = -\Phi_y$
$U = 20\ \mathrm{ms}^{-1}$, $f = 10^{-4}\mathrm{s}^{-1}$, $\Phi = gH$, $H \approx 3$ km.

$$u_t + Uu_x + \phi_x - fv = 0$$

$$v_t + Uv_x + fu = 0$$

$$\phi_t + U\phi_x + \Phi u_x - fUv = 0$$

PDE system discretized by finite differences, periodic B. C.
$\mathbf{H}_k$: observations at synoptic times, over land only.



Ghil *et al*. (1981), Cohn & Dee (Ph.D. theses, 1982 & 1983), etc.

Conventional network

Relative weight of observational *vs.* model errors

$P_\infty = QR/[Q + (1 - \Psi^2)R]$

(a) $Q = 0 \Rightarrow P_\infty = 0$

(b) $Q \neq 0 \Rightarrow$ (i), (ii) and (iii):

(i) "good" observations

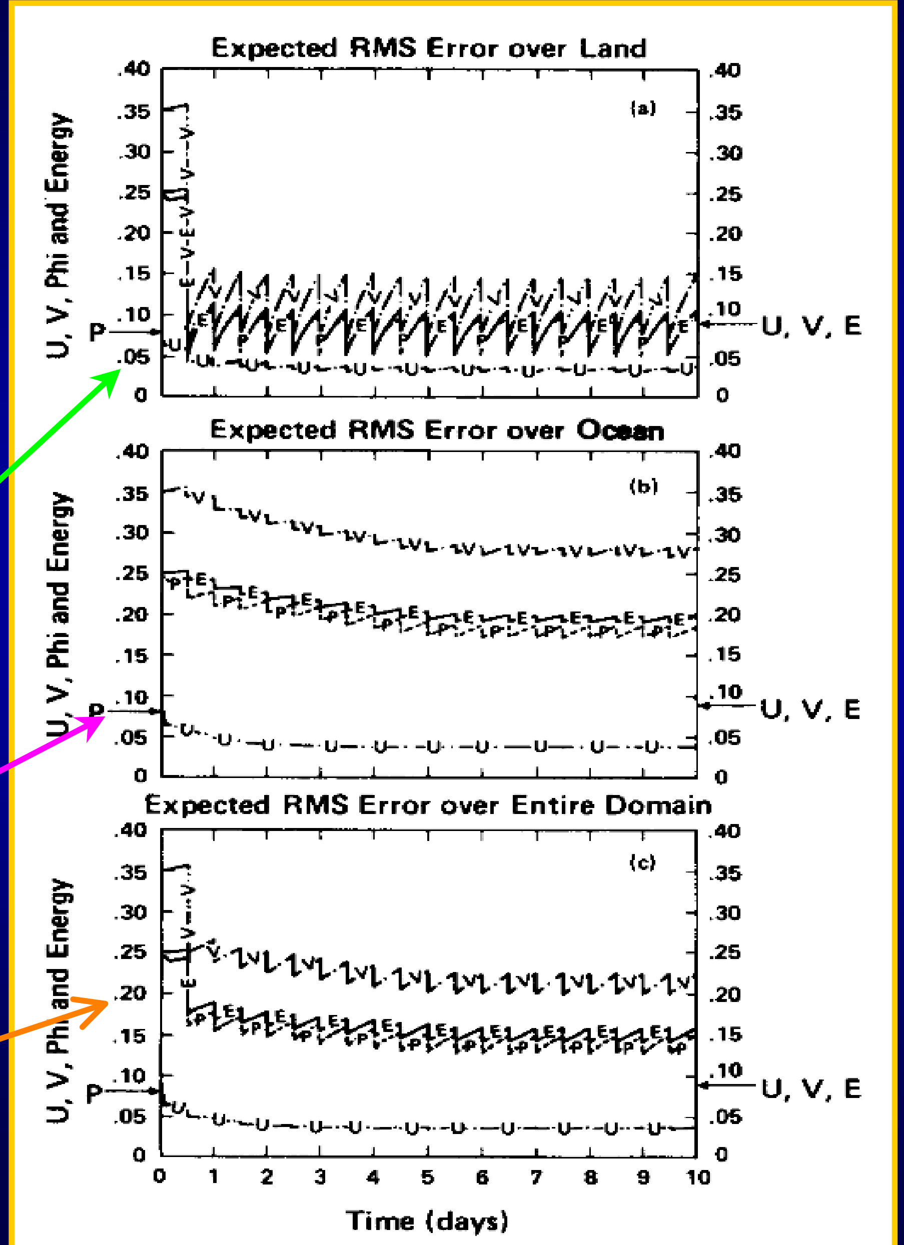
$R << Q \Rightarrow P_\infty \approx R$;

(ii) "poor" observations

$R >> Q \Rightarrow P_\infty \approx Q/(1 - \Psi^2)$;

(iii) always (provided $\Psi^2 < 1$)

$P_\infty \leq \min\{R, Q/(1 - \Psi^2)\}$.



Advection of information

Upper panel (NoSat):

Errors advected off the ocean

ϕ_{300}

Lower panel (Sat):

Errors drastically reduced, as info. now comes in, off the ocean

ϕ_{300}

Halem, Kalnay, Baker & Atlas

(*Bull. Amer. Meteorol. Soc.*, 1982)

{6h fcst} – {conventional (NoSat)}

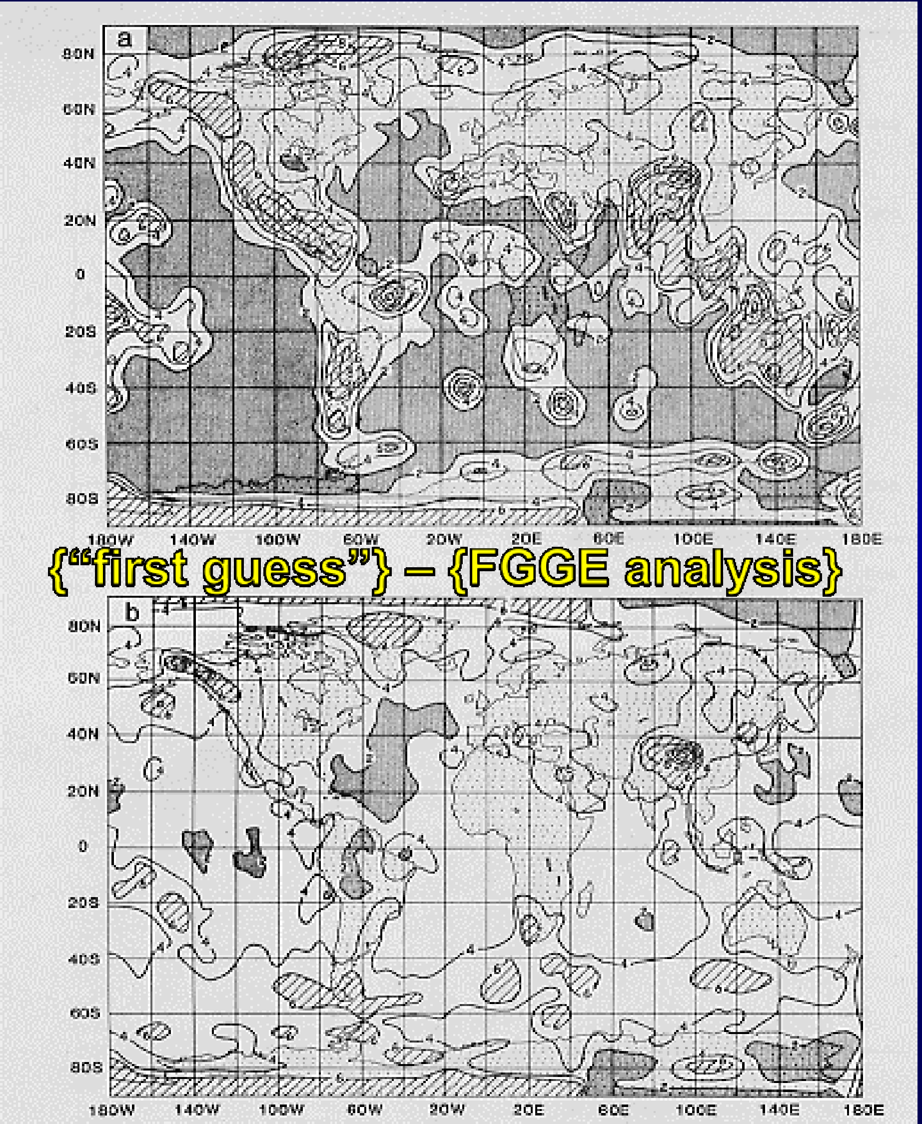


FIG. 5. The rms difference between the 6 h forecast of the 300 mb geopotential height field and the analysis for the period 5–21 January 1979. Contour interval is 20 m. a) Rms difference between the NOSAT analysis and forecast. b) Rms difference between the FGGE analysis and forecast.

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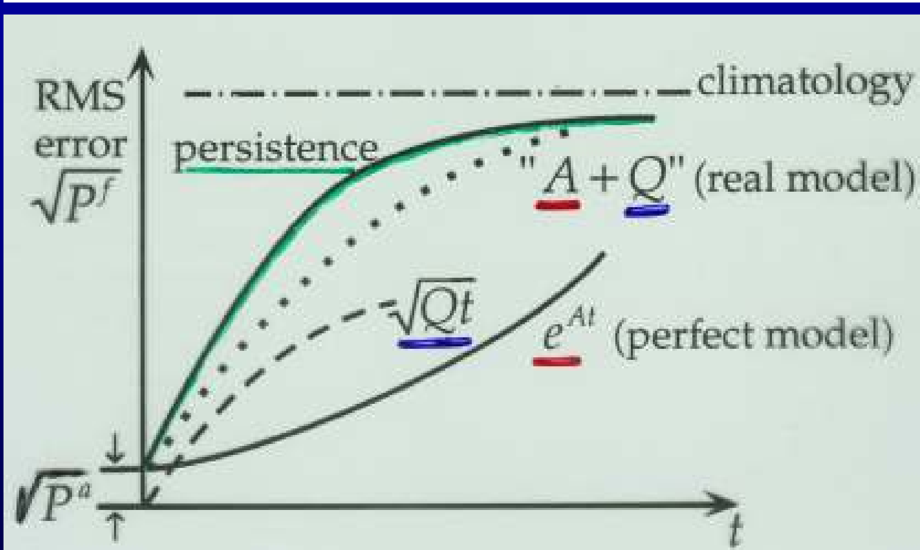
Error components in forecast–analysis cycle

$$\underbrace{P^f}_{\text{first-guess error}} \cong \underbrace{P^a}_{\text{analysis error}} + \Delta t(\underbrace{2AP^a}_{\text{id. twins error growth}} + \underbrace{Q}_{\text{modeling error}})$$

$$(\Psi = e^{A\Delta t} \cong \underline{1 + A\Delta t})$$



The relative contributions to error growth of

- **analysis error**
- **intrinsic error growth**
- **modeling error (stochastic?)**

Assimilation of observations: Stability considerations

Free-System Dynamics (sequential-discrete formulation): ***Standard breeding***

Forecast state:

model integration from a previous analysis

$$\mathbf{x}_{n+1}^{f} = M(\mathbf{x}_{n}^{a})$$

Corresponding perturbative (tangent linear) equation

$$\delta\mathbf{x}_{n+1}^{f} = \mathbf{M}\delta\mathbf{x}_{n}^{a}$$

Observationally Forced System Dynamics (sequential-discrete formulation): ***BDAS***

If observations are available and we assimilate them:

Evolutive equation of the system, subject to forcing by the assimilated data

$$\mathbf{x}_{n+1}^{a} = [\mathbf{I} - \mathbf{K}H \circ] M(\mathbf{x}_{n}^{a}) + \mathbf{K}\mathbf{y}_{n+1}^{o}$$

Corresponding perturbative (tangent linear) equation, if the same observations are assimilated in the perturbed trajectories as in the control solution

$$\delta\mathbf{x}_{n+1}^{a} = [\mathbf{I} - \mathbf{K}\mathbf{H}]\mathbf{M}\delta\mathbf{x}_{n}^{a}$$

- The matrix (**I – KH**) is expected, in general, to have a stabilizing effect (Ghil *et* al., 1981);
- The free-system instabilities, which dominate the error growth during the forecast step , can be reduced during the analysis step.

Carrassi, Ghil, Trevisan & Uboldi (CHAOS, 2008)

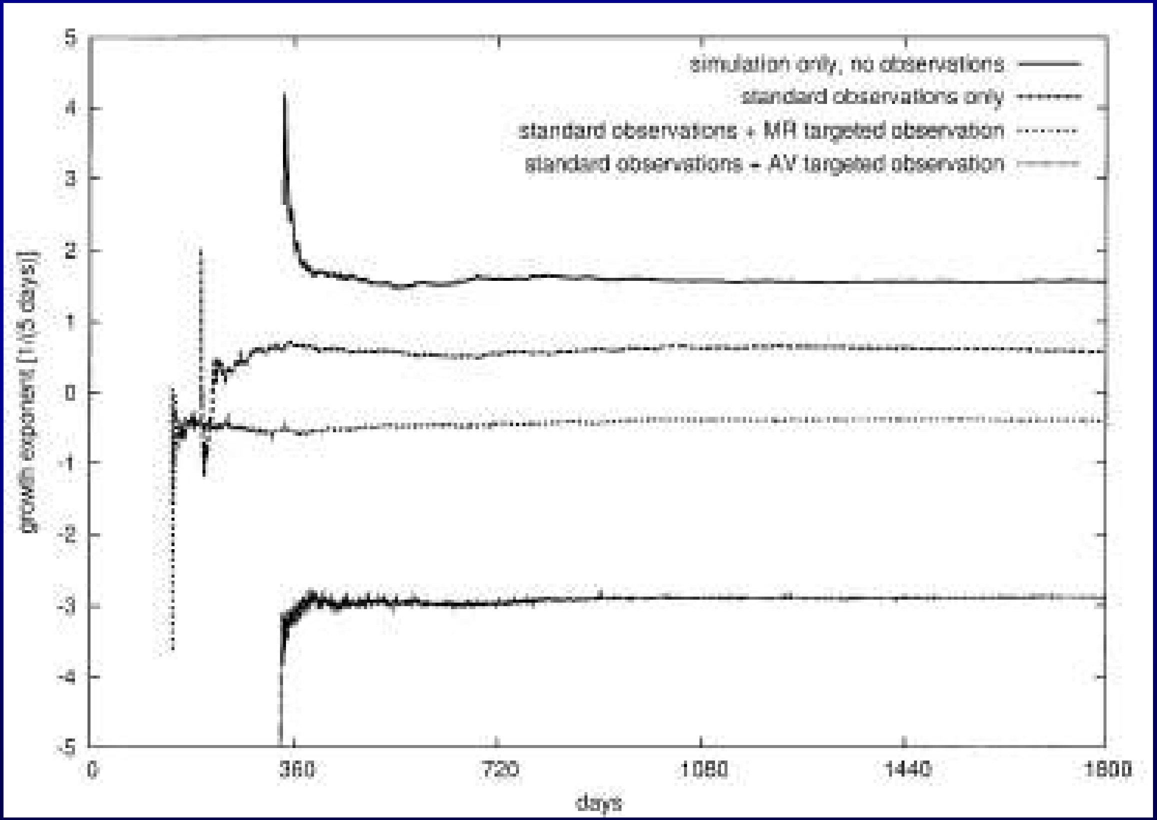
Stabilization of the forecast–assimilation system – I

Assimilation experiment with a low-order chaotic model

- Periodic 40-variable Lorenz (1996) model;
- Assimilation algorithms: replacement (Trevisan & Uboldi, 2004), replacement + one adaptive obs'n located by multiple replication (Lorenz, 1996), replacement + one adaptive obs'n located by BDAS and assimilated by AUS (Trevisan & Uboldi, 2004).

BDAS: Breeding on the Data Assimilation System

AUS: Assimilation in the Unstable Subspace



Trevisan & Uboldi (*J. Atmos. Sci.*, 2004)

Stabilization of the forecast–assimilation system – II

Assimilation experiment with the 40-variable Lorenz (1996) model

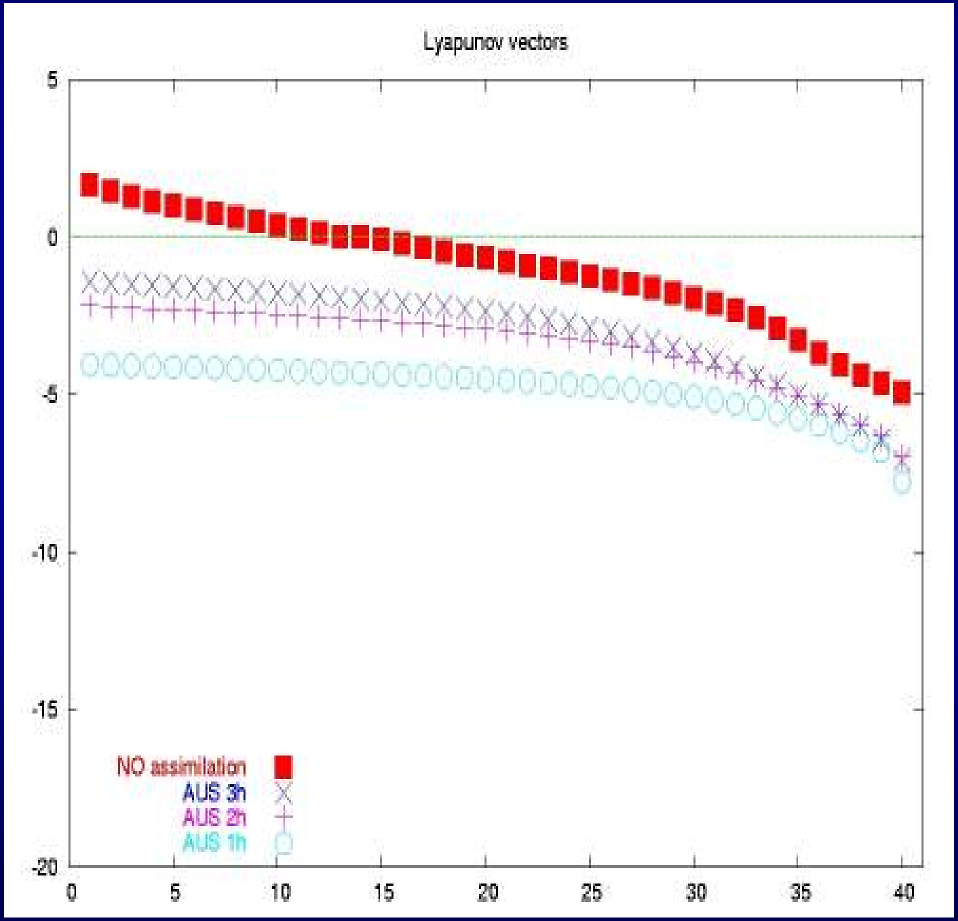
Spectrum of Lyapunov exponents:

Red: free system

Dark blue: AUS with 3-hr updates

Purple: AUS with 2-hr updates

Light blue: AUS with 1-hr updates



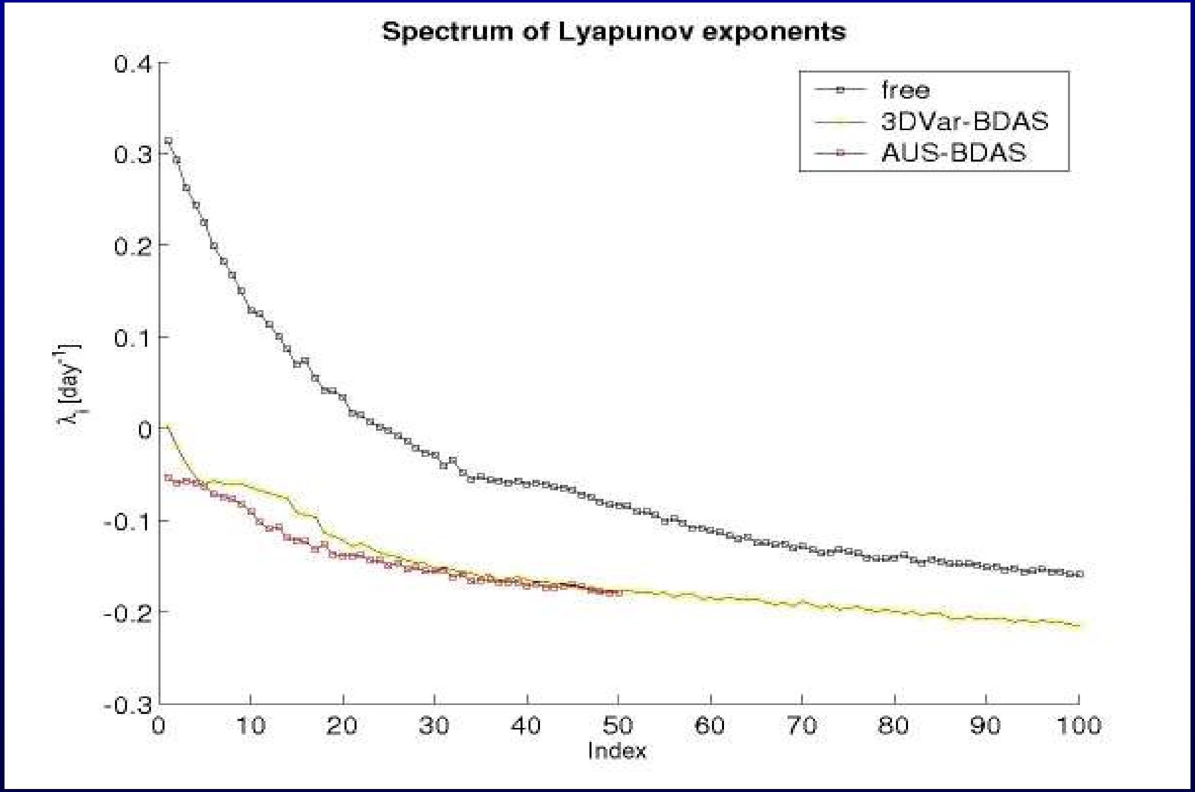
Carrassi, Ghil, Trevisan & Uboldi, (*CHAOS*, 2008)

Stabilization of the forecast–assimilation system – III

Assimilation experiment with an intermediate atmospheric circulation model

- 64-longitudinal x 32-latitudinal x 5 levels periodic channel QG-model (Rotunno & Bao, 1996)
- Perfect-model assumption
- Assimilation algorithms: 3-DVar (Morss, 2001); AUS (Uboldi *et al.*, 2005; Carrassi *et al.*, 2006)

Observational forcing ⇒ Unstable subspace reduction



↗ Free System

Leading exponent:

$\lambda_{max} \approx 0.31$ days^{-1};

Doubling time ≈ 2.2 days;

Number of positive exponents:

$N^+ = 24$;

Kaplan-Yorke dimension ≈ 65.02.

↗ 3-DVar–BDAS

Leading exponent:

$\lambda_{max} \approx 0.002$ days^{-1};

Kaplan-Yorke dimension ≈ 1.1

↗ AUS–BDAS

Leading exponent:

$\lambda_{max} \approx -0.52\text{x}10^{-3}$ days^{-1}

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Parameter Estimation

a) Dynamical model

$dx/dt = M(x, \mu) + \eta(t)$

$y^o = H(x) + \varepsilon(t)$

Simple (EKF) idea – augmented state vector

$d\mu/dt = 0$, $X = (x^T, \mu^T)^T$

b) Statistical model

$L(\rho)\eta = w(t)$, L – AR(MA) model, $\rho = (\rho_1, \rho_2, \ldots. \rho_M)$

Examples: 1) Dee *et al*. (*IEEE*, 1985) – estimate a few parameters in the covariance matrix $Q = E(\eta, \eta^T)$; also the bias $\langle\eta\rangle = E\eta$;

2) POPs - Hasselmann (1982, Tellus); Penland (1989, *MWR*; 1996, *Physica D*); Penland & Ghil (1993, *MWR*)

3) $dx/dt = M(x, \mu) + \eta$: Estimate both M & Q from data (Dee, 1995, *QJ*), Nonlinear approach: Empirical mode reduction (EMR: Kravtsov *et al*., *J. Clim*., 2005; Kondrashov *et al*., *J. Clim*., 2005, *J. Atmos. Sci*., 2006; Kravtsov *et al*., in Palmer & Williams (Eds.), Cambridge U. P., 2010; Strounine *et al*., *Physica D*, 2010)

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Sequential parameter estimation

- "**State augmentation**" method – uncertain parameters are treated as additional state variables.
- Example: one unknown parameter μ

$$\bar{x}_k = \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} = \begin{pmatrix} F(x_{k-1}, \mu_{k-1}) \\ \mu_{k-1} \end{pmatrix} + \begin{pmatrix} \epsilon_k \\ \epsilon^{\mu}_{k-1} \end{pmatrix}$$

$$y^o_k = \begin{pmatrix} H & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} + \epsilon^0 = \bar{H}\bar{x}_k + \epsilon^0$$

$$\bar{x}^a_k = \bar{x}^f_k + \bar{K}(y^o_k - \bar{H}\bar{x}^f_k); \quad \bar{K} = \bar{P}^f \bar{H}^T (\bar{H}\bar{P}^f \bar{H}^T + R)^{-1}$$

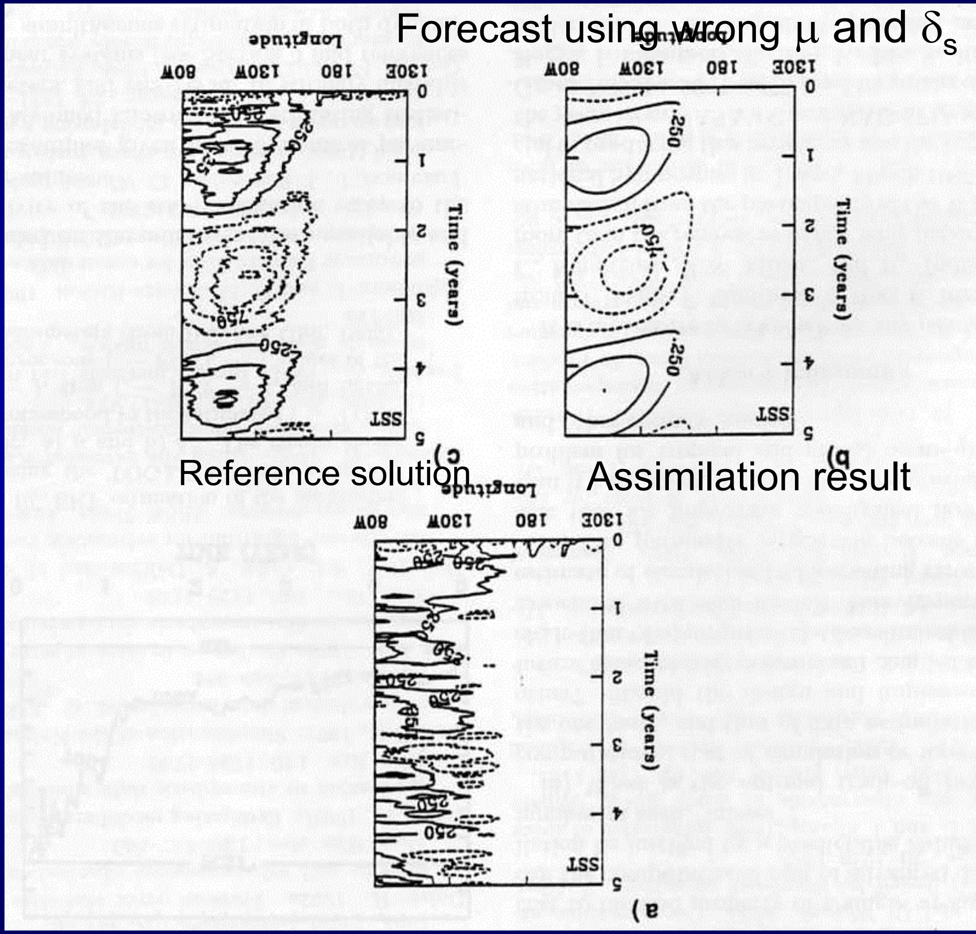
- **The parameters are not directly observable, but** the **cross-covariances** drive parameter changes from innovations of the state:

$$\bar{P}^f = \begin{pmatrix} P^f_{xx} & P^f_{x\mu} \\ P^f_{\mu x} & P^f_{\mu\mu} \end{pmatrix}; \quad \bar{K} = \begin{pmatrix} P^f_{xx} H^T \\ P^f_{\mu x} H^T \end{pmatrix} (H P^f_{xx} H^T + R)^{-1}$$

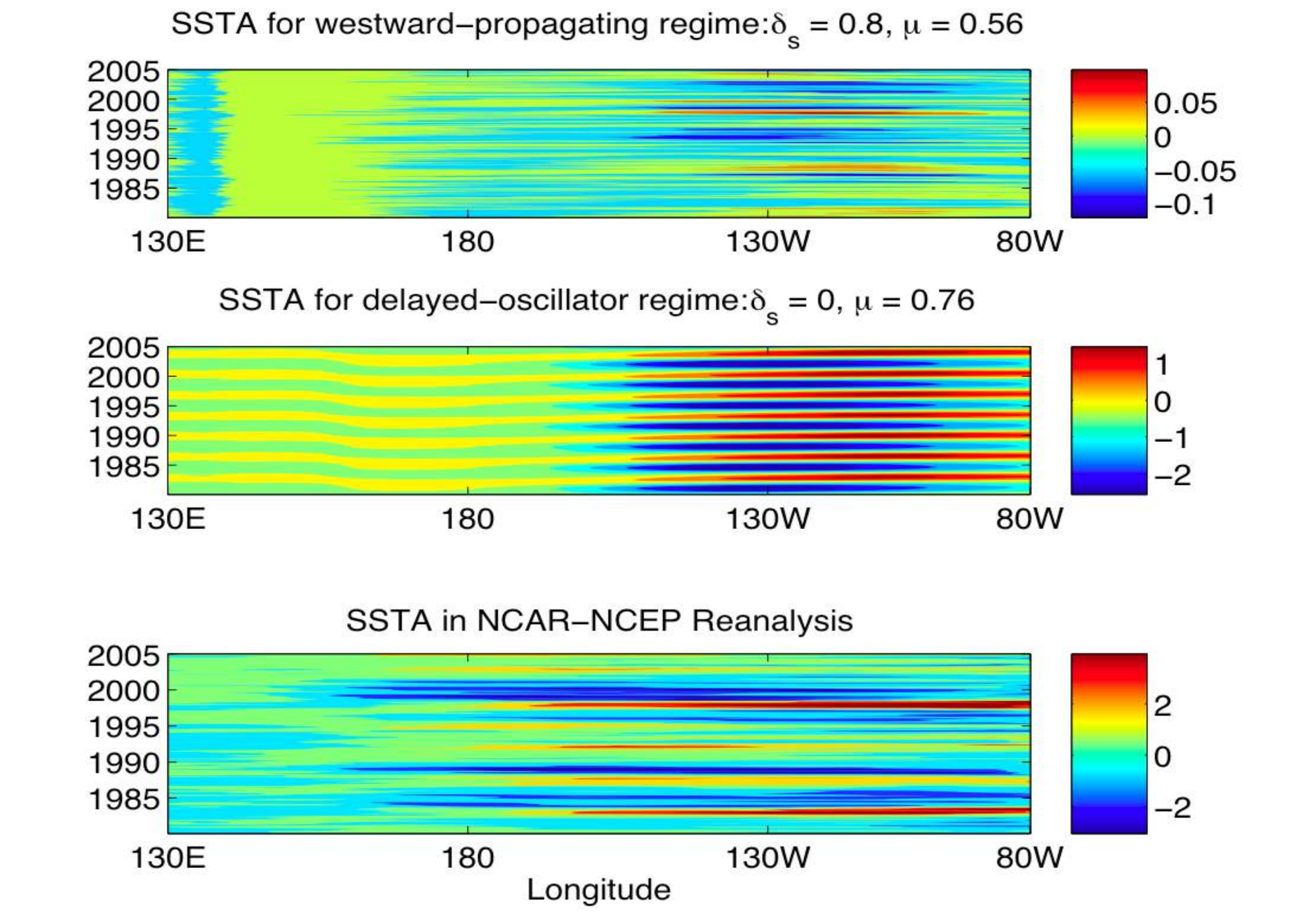
- Parameter estimation is always a **nonlinear problem,** even if the model is **linear** in terms of the model state: use **Extended Kalman Filter (EKF).**

Parameter estimation for coupled O-A system

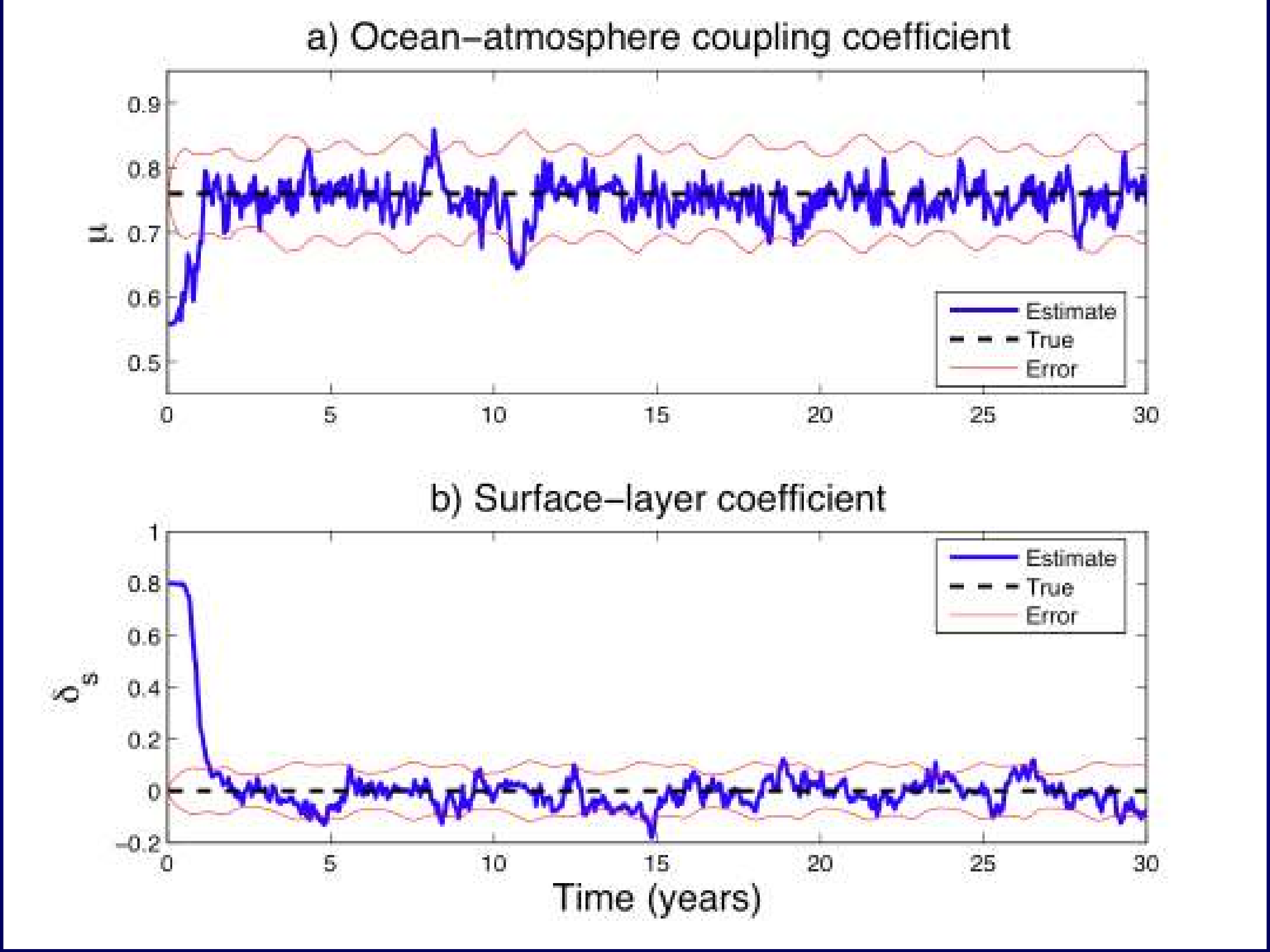
- Intermediate coupled model (ICM: Jin & Neelin, *JAS*, 1993)
- Estimate the state vector $W = (T', h, u, v)$, along with the coupling parameter μ and surface-layer coefficient δ_s by assimilating data from a single meridional section.
- The ICM model has errors in its initial state, in the wind stress forcing & in the parameters.
- Hao & Ghil (1995, *Proc. WMO Symp. DA Tokyo*); Ghil (1997, *JMSJ*); Sun *et al*. (2002, *MWR*).
- Kondrashov, Sun & Ghil (*Monthly Weather Rev.*, 2008)



Coupled O-A Model (ICM) vs. Observations

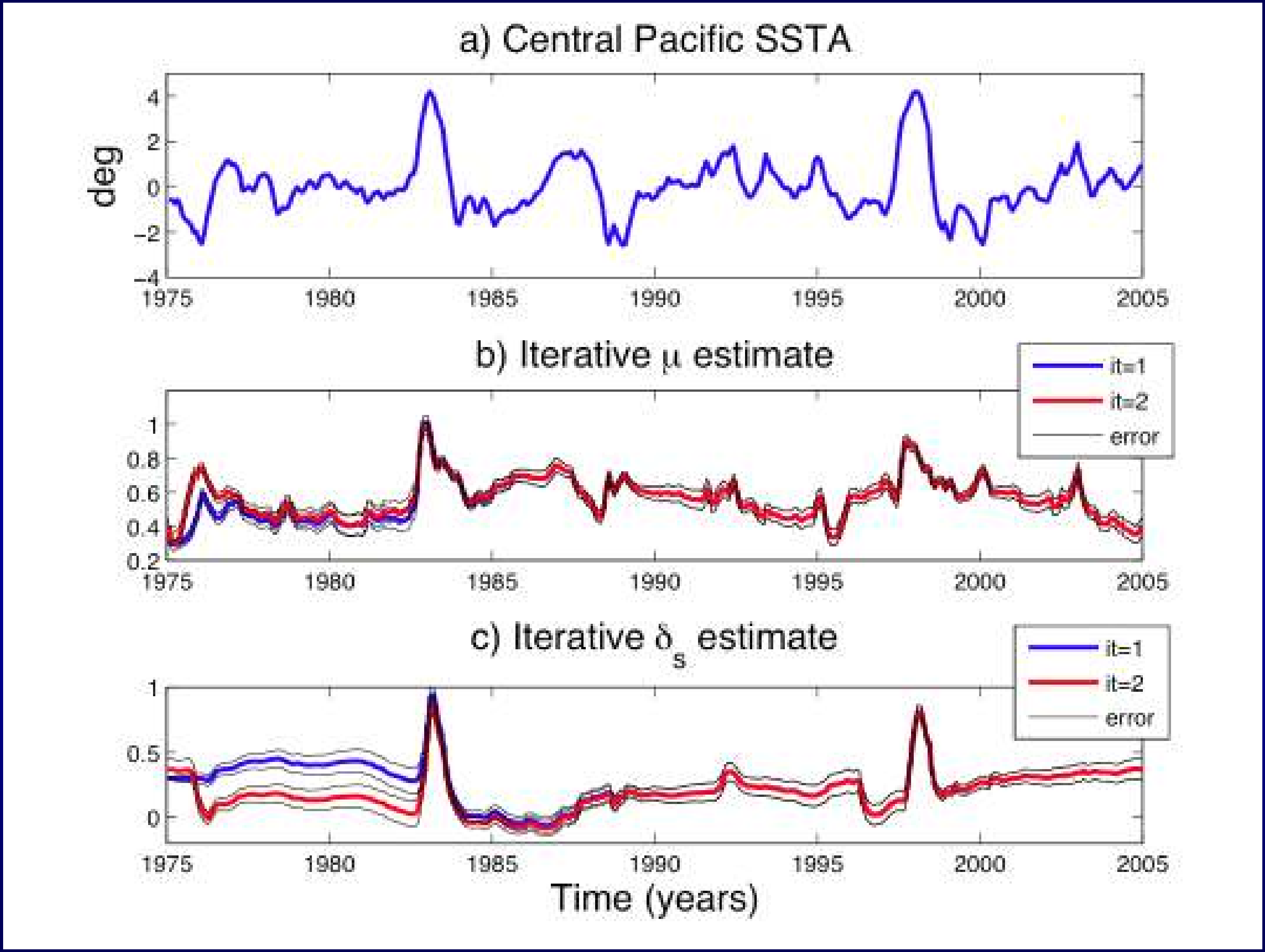


Convergence of Parameter Values – I



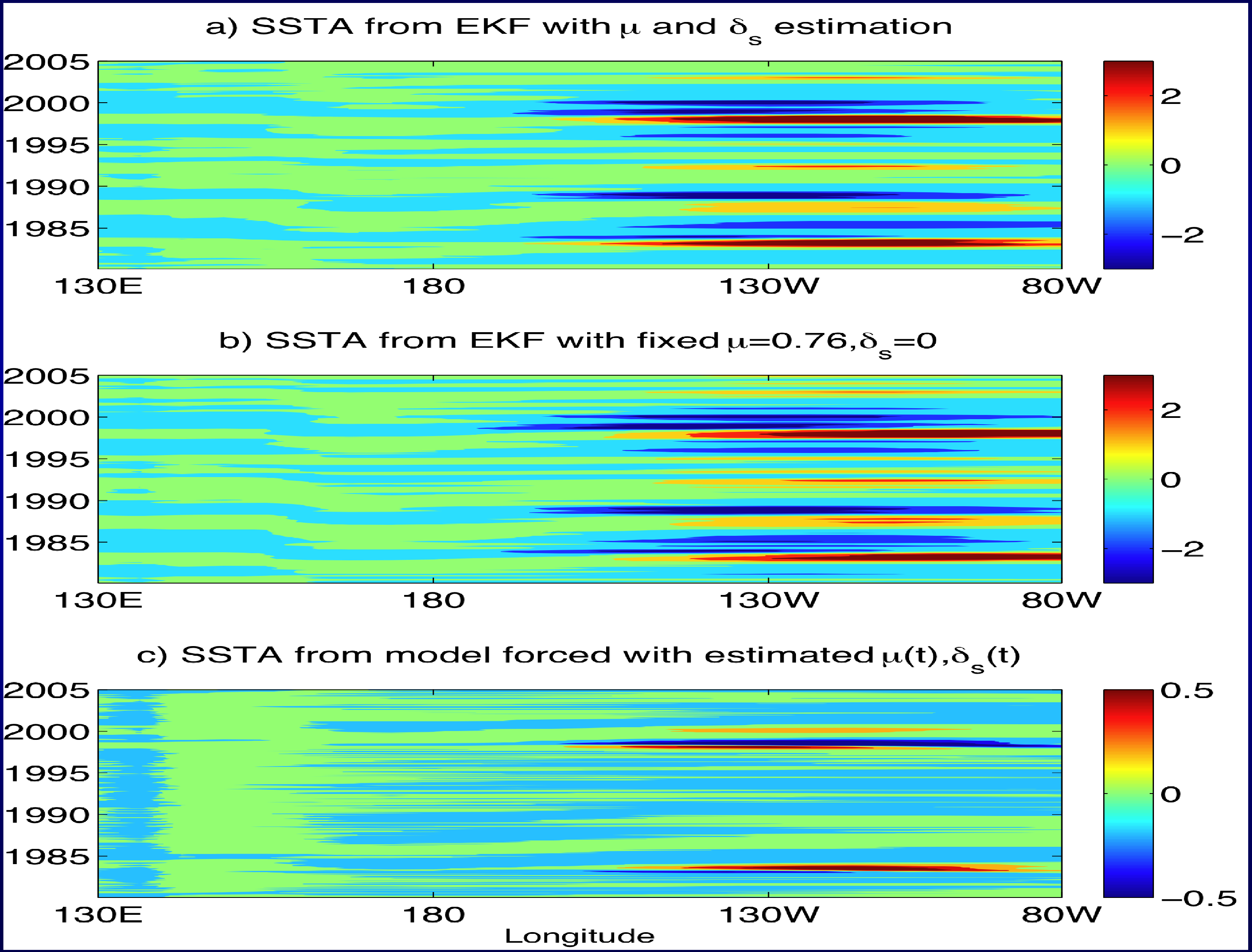
Identical-twin experiments

Convergence of Parameter Values – II



Real SST anomaly (SSTA) data

EKF results with and w/o parameter estimation



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Computational advances

a) Hardware

- more computing power (CPU throughput)
- larger & faster memory (3-tier)

b) Software

- better numerical implementations of algorithms
- automatic adjoints
- block-banded, reduced-rank & other sparse-matrix algorithms
- better ensemble filters
- efficient parallelization, ….

How much DA *vs*. forecast?

- Design integrated observing–forecast–assimilation systems!

Observing system design

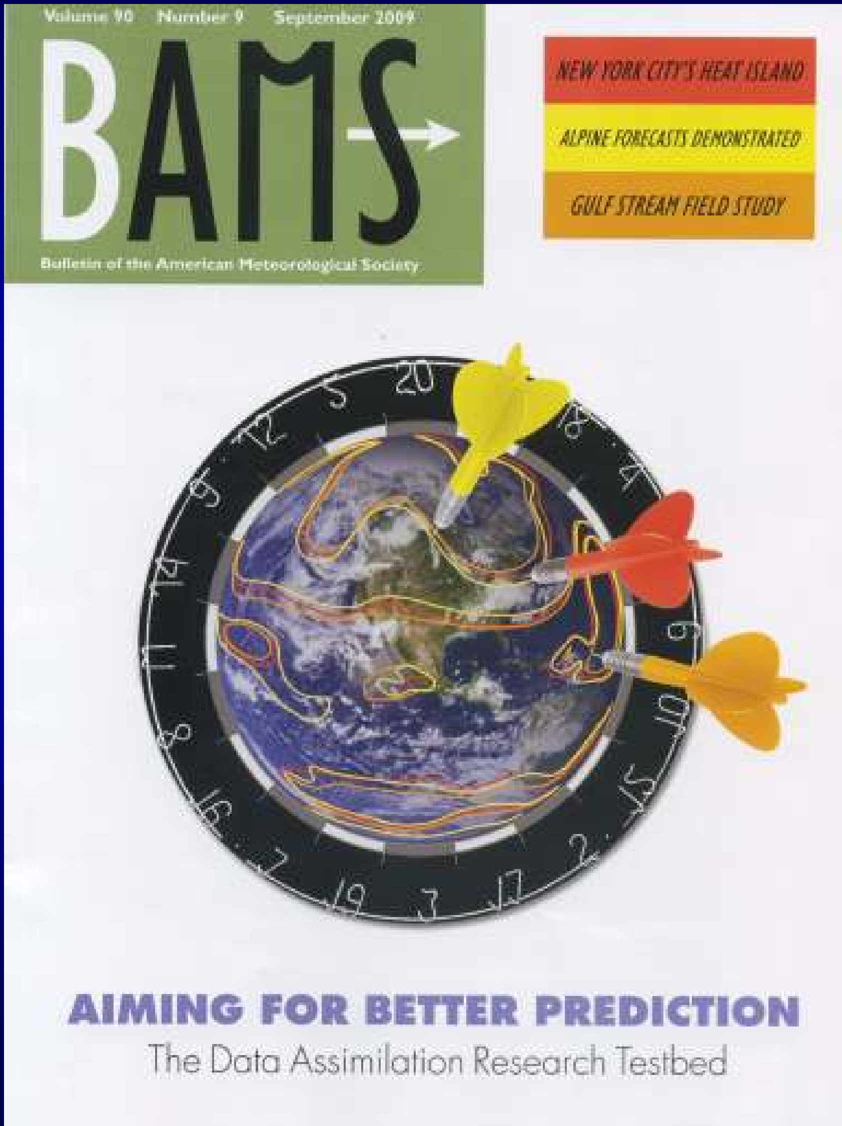
- ↗ **Need no more (independent) observations than *d-o-f* to be tracked:**
  - "features" (Ide & Ghil, *Dyn. Atmos. Oceans,* 1997a, b);
  - instabilities (Todling & Ghil, 1994 + Ghil & Todling, 1996, *MWR*);
  - trade-off between mass & velocity field (Jiang & Ghil, *JPO*, 1993).
- ↗ The cost of **advanced** DA is **much less** than that of instruments & platforms:
  - at best use DA **instead** of instruments & platforms.
  - at worst use DA to determine which instruments & platforms (**advanced** OSSE)
- ↗ **Use any** observations, if forward modeling is possible (observing operator H)
  - satellite images, 4-D observations;
  - pattern recognition in observations and in phase-space statistics.

Conclusions

- Theoretical concepts can play a useful role in devising better practical algorithms, and vice-versa.
- Judicious choices of observations and method can stabilize the forecast-assimilation cycle.
- Trade-off between cost of observations and of data assimilation.

- *Assimilation of ocean data in the coupled O–A system is useful.*
- *They help estimate both ocean and coupling parameters.*
- *Changes in estimated parameters compensate for model imperfections.*

DA Research Testbed (DART)



ARTICLES

THE DATA ASSIMILATION RESEARCH TESTBED

A Community Facility

BY JEFFREY ANDERSON, TIM HOAR, KEVIN RAEDER, HUI LIU, NANCY COLLINS, RYAN TORN, AND AVELINO ARELLANO

DART, developed and maintained at the National Center for Atmospheric Research, provides well-documented software tools for data assimilation education, research, and development.

Data assimilation combines observations with model forecasts to estimate the state of a physical system. Developed in the 1960s (Daley 1991; Kalnay 2003) to provide initial conditions for numerical weather prediction (NWP; Lynch 2006), data assimilation can do much more than initialize forecasts. Repeating the NWP process after the fact using all available observations and state-of-the-art data assimilation produces reanalyses, the best available estimate of the atmospheric state (Kistler et al. 2001; Uppala et al. 2005; Compo et al. 2006). Data assimilation can estimate the value of existing or hypothetical observations (Khare and Anderson 2006a; Zhang et al. 2004). Applications include predicting efficient flight paths for planes that release dropsondes (Bishop et al. 2001) and assessing the potential impact of a new satellite instrument before it is built or launched (Mourre et al. 2006). Data assimilation tools can also be used to evaluate forecast models, identifying quantities that are poorly predicted and comparing models to assess relative strengths and weaknesses. Data assimilation can guide model development by estimating values for model parameters that are most consistent with observations (Houtekamer et al. 1996; Aksoy et al. 2006). Assimilation is now used also for the ocean (Keppenne and Rienecker 2002; Zhang et al. 2005), land surface (Reichle et al. 2002), cryosphere (Stark et al. 2008), biosphere (Williams et al. 2004), and chemical constituents (Constantinescu et al. 2007). Assimilation tools under different names are used in other areas of geophysics, engineering, economics, and social sciences.

The Data Assimilation Research Testbed (DART) is an open-source community facility that provides software tools for data assimilation research.

AFFILIATIONS: Anderson, Hoar, Raeder, Liu, Collins—NCAR* Data Assimilation Research Section, Boulder, Colorado; Torn—Department of Earth and Atmospheric Sciences, University at Albany, State University of New York, Albany, New York; Arellano—NCAR Atmospheric Chemistry Division, Boulder, Colorado
*The National Center for Atmospheric Research is sponsored by the National Science Foundation
CORRESPONDING AUTHOR: Jeffrey Anderson, NCAR, P.O. Box 3000, Boulder, CO 80307-3000
E-mail: jla@ucar.edu

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"Miss Peterson, may I go home? I can't assimilate any more data today."

J.B. Handelsman (5/31/1969)

Reserve slides

Estimating noise – I

$Q_1 = Q_{slow}$, $Q_2 = Q_{fast}$, $Q_3 = 0$;

$R_1 = 0$, $R_2 = 0$, $R_3 = R$;

$Q = \sum \alpha_i Q_i$; $R = \sum \alpha_i R_i$;

$\alpha(0) = (6.0, 4.0, 4.5)^T$;

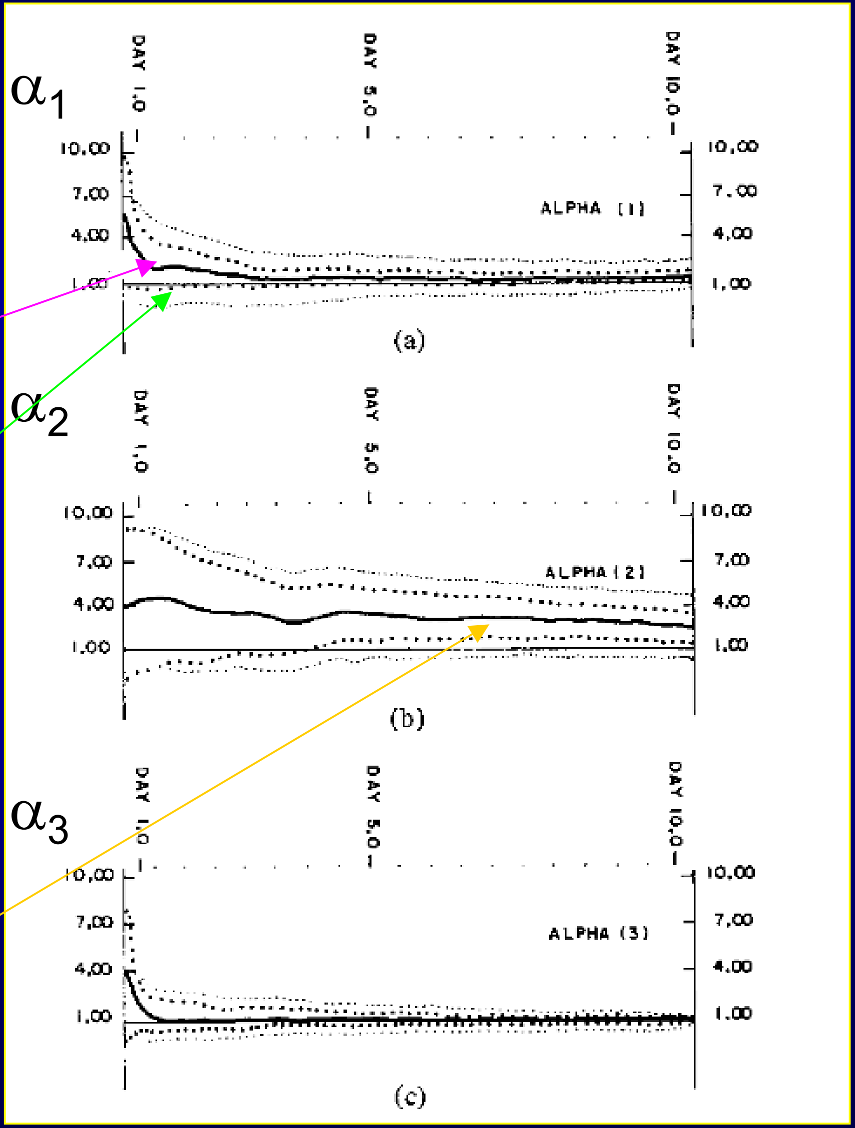
$Q(0) = 25*I$.

estimated

true (α =1)

Dee *et al*. (1985, *IEEE Trans. Autom. Control*, **AC-30**)

Poor convergence for Q_{fast}?



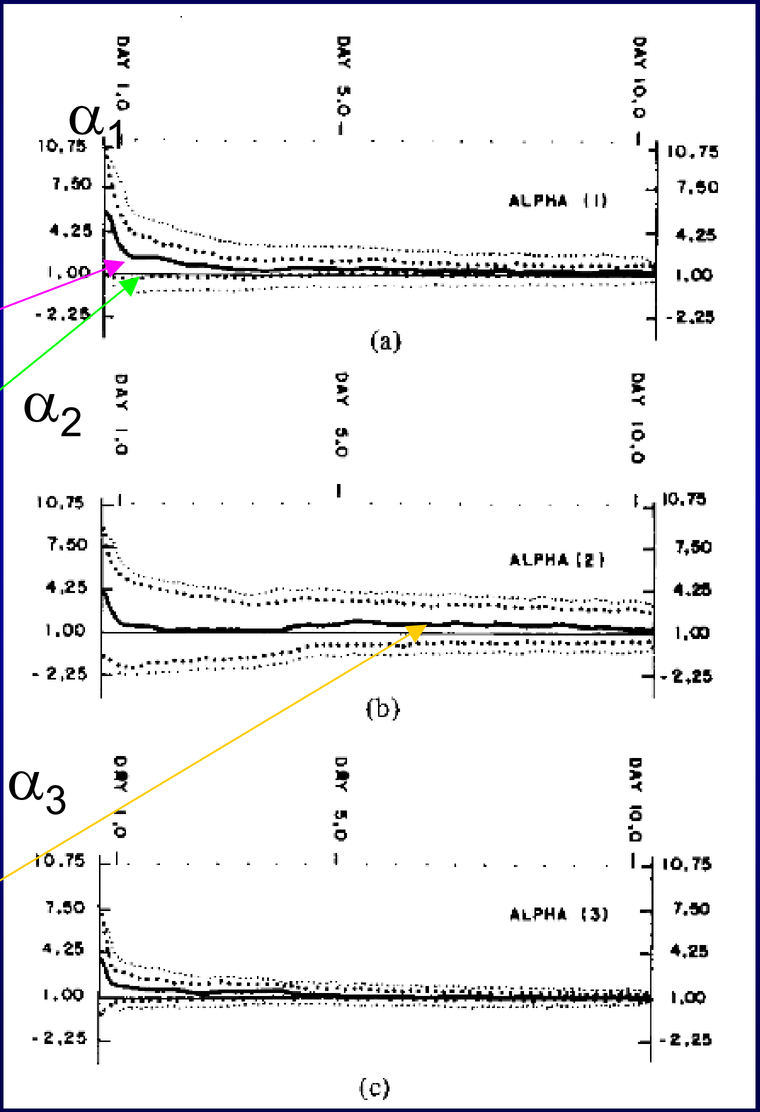
Estimating noise – II

Same choice of $\alpha(0)$, Q_i , and R_i but

$$\Theta(0) = 25 * \begin{bmatrix} 1 & 0.8 & 0 \\ 0.8 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

estimated

true (α = 1)



Dee *et al.* (1985, *IEEE Trans. Autom. Control*, **AC-30**)

Good convergence for Q_{fast}!

Evolution of DA – I

TABLE I. CHARACTERISTICS OF DATA ASSIMILATION SCHEMES IN OPERATIONAL USE AT THE END OF THE 1970s[a]

Organization or country	Operational analysis methods	Analysis area	Analysis/forecast
Australia	Successive correction method (SCM)	SH[d]	12 hr
	Variational blending techniques	Regional	6 hr
Canada	Multivariate 3-D statistical interpolation	NH[d] Regional	6 hr (3 hr for the surface)
France	SCM; wind-field and mass-field balance through first guess	NH	6 hr
	Multivariate 3-D statistical interpolation	Regional	
F.R. Germany	SCM. Upper-air analyses were built up, level by level, from the surface	NH	12 hr (6 hr for the surface)
	Variational height/wind adjustment		Climatology only as preliminary fields
Japan	SCM	NH	12 hr
	Height-field analyses were corrected by wind analyses	Regional	
Sweden	Univariate 3-D statistical interpolation	NH	12 hr
	Variational height/wind adjustment	Regional	3 hr
United Kingdom	Hemispheric orthogonal polynomial method		
	Univariate statistical interpolation (repeated insertion of data)	Global	6 hr
U.S.A.	Spectral 3-D analysis	Global	
	Multivariate 3-D statistical interpolation	Global	6 hr
U.S.S.R.	2-D[c] statistical interpolation	NH	12 hr
ECMWF[b]	Multivariate 3-D statistical interpolation	Global	6 hr

[a] After Gustafsson (1981).
[b] European Centre for Medium Range Weather Forecasts.
[c] 2-D is in a horizontal plane.
[d] Southern Hemisphere and Northern Hemisphere, respectively.

Transition from "early" to "mature" phase of DA in NWP:

- no Kalman filter (Ghil *et al*., 1981(*))
- no adjoint (Lewis & Derber, *Tellus*, 1985);
  Le Dimet & Talagrand (*Tellus*, 1986)

(*) Bengtsson, Ghil & Källén (Eds., 1981), *Dynamic Meteorology: Data Assimilation Methods.*

M. Ghil & P. M.-Rizzoli (*Adv. Geophys.*, 1991).

Evolution of DA – II

Table IV. Duality Relationships Between Stochastic Estimation and Deterministic Control[a]

A. Continuous (linear) Kalman Filter

System Model	$\dot{\mathbf{w}}^{t}(t) = F(t)\mathbf{w}^{t}(t) + G(t)\mathbf{b}^{t}(t), \quad \mathbf{b}^{t}(t) \sim \mathrm{N}[0, Q(t)]$
Measurement Model	$\mathbf{w}^{0}(t) = H(t)\mathbf{w}^{t}(t) + \mathbf{b}^{0}(t), \quad \mathbf{b}^{0}(t) \sim \mathrm{N}[0, R(t)]$
State estimation	$\dot{\mathbf{w}}^{a}(t) = F(t)\mathbf{w}^{a}(t) + K(t)[\mathbf{w}^{0}(t) - H(t)\mathbf{w}^{a}(t)], \quad \mathbf{w}^{a}(0) = \mathbf{w}^{a}_{0}$
Error covariance propagation (Riccati Equation)	$\dot{P}(t) = F(t)P(t) + P(t)F^{T}(t) + G(t)Q(t)G^{T}(t) - K(t)R(t)K^{T}(t), \quad P(0) = P_0$
Kalman Gain	$K(t) = P(t)H^{T}(t)R^{-1}(t)$
Initial conditions	$E[\mathbf{w}^{t}(0)] = \mathbf{w}^{a}_{0}, \quad E\{[\mathbf{w}^{t}(0) - \mathbf{w}^{a}_{0}][\mathbf{w}^{t}(0) - \mathbf{w}^{a}_{0}]^{T}\} = P_0$
Assumptions	$R^{-1}(t)$ exists $E\{\mathbf{b}^{t}(t)[\mathbf{b}^{0}(t')]^{T}\} = 0$
Performance Index	$p^{f,a}(t) = E\{[\mathbf{w}^{f,a} - \mathbf{w}^{t}][\mathbf{w}^{f,a} - \mathbf{w}^{t}]^{T}\}$

B. Continuous (linear) Optimal Control

System Model	$\dot{\mathbf{w}}^{t}(t) = \tilde{F}(t)\mathbf{w}(t) + \tilde{H}(t)\mathbf{u}(t)$
Measurement Model	$\mathbf{w}^{0}(t) = \mathbf{w}(t)$ (all system variables are measured)
Performing control	$\mathbf{u}(t) = -\tilde{K}(t)\mathbf{w}(t)$
Performance propagation (Riccati Equation)	$\tilde{P}(t) = -\tilde{F}^{T}(t)\tilde{P}(t) - \tilde{P}(t)\tilde{F}(t) - \tilde{Q}(t) + \tilde{P}(t)\tilde{H}(t)\tilde{K}(t)$
Control Gain	$\tilde{K}(t) = \tilde{R}^{-1}(t)\tilde{H}(t)\tilde{P}(t)$
Terminal conditions	$\mathbf{w}(t_f) = 0$ $\mathbf{P}(t_f) = \tilde{Q}_f$
Cost function	$J[\mathbf{w}, \mathbf{u}] = \mathbf{w}_f^{T}\tilde{Q}_f\mathbf{w}_f + \int_0^{t_f} [\mathbf{w}^{T}(t)\tilde{Q}(t)\mathbf{w}(t) + \mathbf{u}^{T}(t)\tilde{R}(t)\mathbf{u}(t)]\, dt$

C. Estimation-Control Duality

Estimation	Control
t_0 initial time	t_f final time
$\mathbf{w}(t)$ unobservable state variable of random process	$\mathbf{w}(t)$ observable state variable to be controlled
$\mathbf{w}^{0}(t)$ random observations	$\mathbf{u}(t)$ deterministic control
$F(t)$ dynamic matrix	$\tilde{F}^{T}(t)$ dynamic matrix
$Q(t)$ covariance matrix for the model errors	$\tilde{Q}(t)$ quadratic matrix defining acceptable errors on model variables
$H(t)$ effect of observations on state variables	$\tilde{H}(t)$ effect of control on state variables
$P(t)$ covariance of estimation error under optimization	$\tilde{P}(t)$ quadratic performance under optimization
$K(t)$ weighting on observation for optimal estimation	$\tilde{K}(t)$ weighting on state for optimal control

[a] (A), Kalman filter as the optimal solution for the former problem; (B), optimal solution for the latter problem; (C), equivalences between the two (after Kalman, 1960, and Gelb, 1974, Section 9.5; courtesy of R. Todling).

Cautionary note:

"Pantheistic" view of DA:

- variational ~ KF;
- 3- & 4-D Var ~ 3- & 4-D PSAS or EnKF.

Fashionable to claim it's all the same but it's not:

- **God** is in **everything**,
- **but the devil is in the details.**

M. Ghil & P. M.-Rizzoli (*Adv. Geophys.*, 1991).

The DA Maturity Index of a Field

- **Pre-DA:** few data, poor models
    - The theoretician: Science is truth, don't bother me with the facts!
    - The observer/experimentalist: Don't ruin my beautiful data with your lousy model!!
- **Early DA:**
    - Better data, so-so models.
    - Stick it (the observations) in – direct insertion, nudging.
- **Advanced DA:**
    - Plenty of data, fine models.
    - E(n)KF, 4-D Var (2^{nd} duality).
- **Post-industrial DA:**

(Satellite) images --> (weather) forecasts, climate "movies" …

(Ihler, Kirshner, Ghil, Robertson & Smyth, *Physica D*, 2007)

Overall Conclusion

- No observing system without data assimilation and no assimilation without dynamics[a]
- Quote of the day: "You cannot step into the same river[b] twice[c]" (Heracleitus, *Trans. Basil. Phil. Soc. Miletus*, *cca.* 500 B.C.)

[a] of state and errors

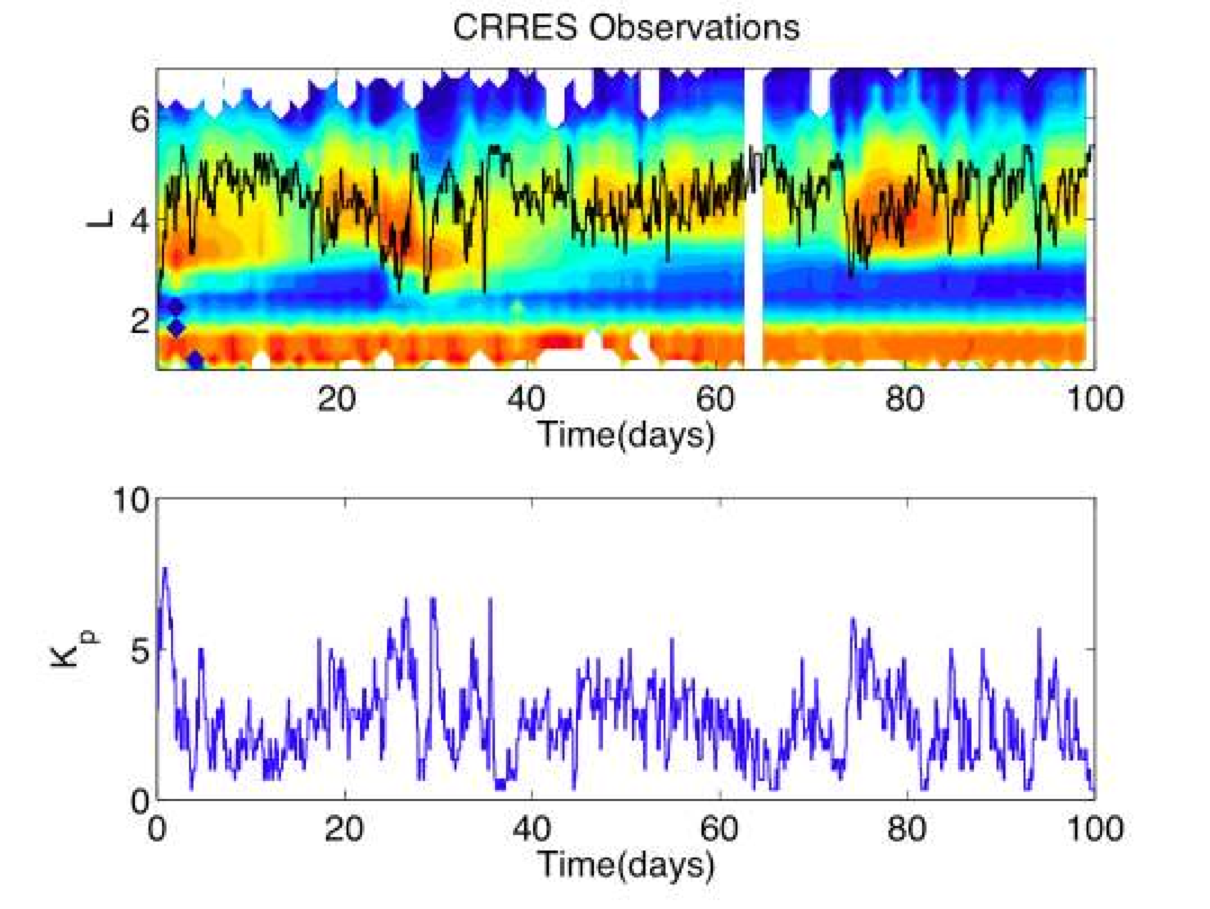
[B] Meandros

[c] *"*You cannot do so even once" (subsequent development of "flux" theory by Plato, c*ca.* 400 B.C.)

Τα παντα ρεει = Everything flows

Parameter Estimation for Space Physics – I

Daily fluxes of 1 MeV relativistic electrons in Earth's outer radiation belt
(CRRES observations from 28 August 1990)
***K*$_p$ - index of solar activity (external forcing) –** used to determine the position of the plasmapause L_{pp} (black) in the observations

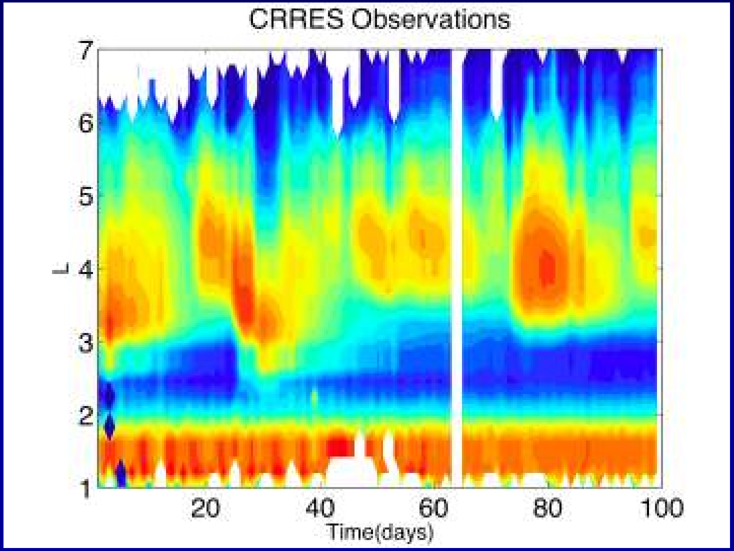


Kondrashov, Shprits, Ghil & Thorne (*J. Geophys. Res.*, 2007)

Parameter estimation for space physics – II

HERRB-1D code (Y. Shprits) –
estimating phase-space density
f and electron lifetime τ_L:

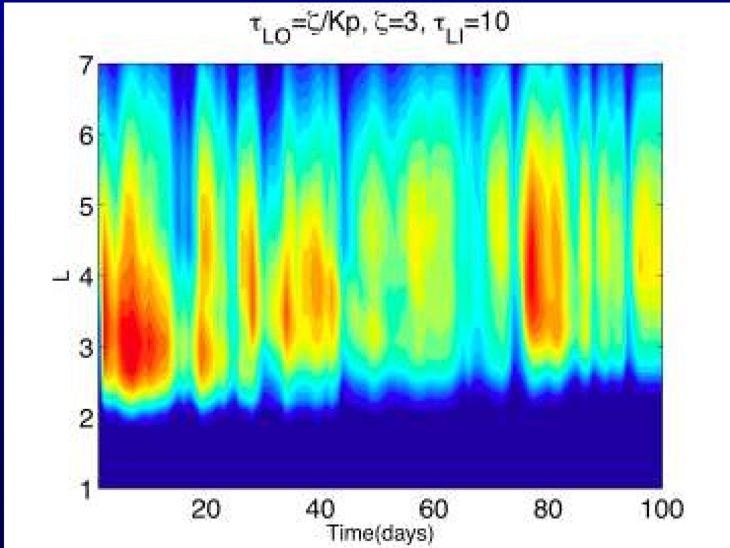
$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left(L^{-2} D_{LL} \frac{\partial f}{\partial L}\right) - \frac{f}{\tau_L}$$

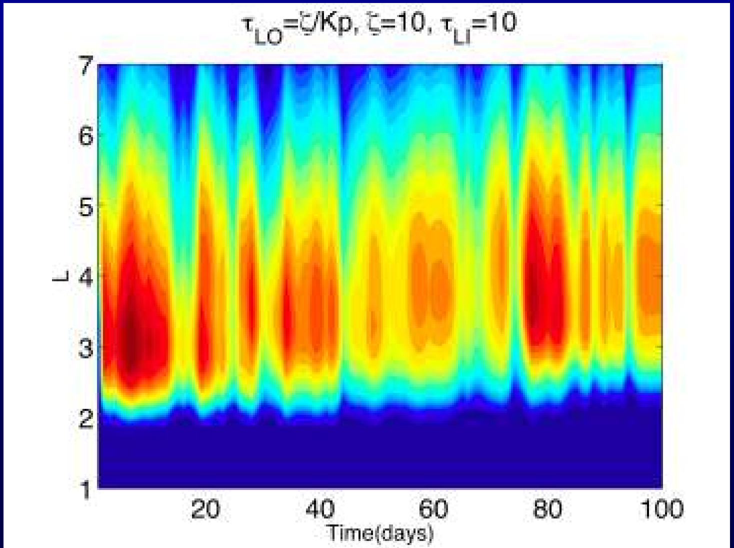


Different lifetime parameterizations for plasmasphere – out/in:

$\tau_{Lo} = \zeta / K_p(t)$; τ_{Li} = const.

What are the **optimal** lifetimes to match the observations best?





Parameter estimation for space physics – III

Daily observations from the "truth" —

$\tau_{Lo} = \zeta/K_p$, $\zeta = 3$, and $\tau_{LI} = 20$ —

are used to correct the model's "wrong" parameters, $\zeta = 10$ and $\tau_{LI} = 10$.

The estimated error $tr(P_f)$ —> actual.

When the parameters' assumed uncertainty is large enough, their EKF estimates converge rapidly to the "truth."