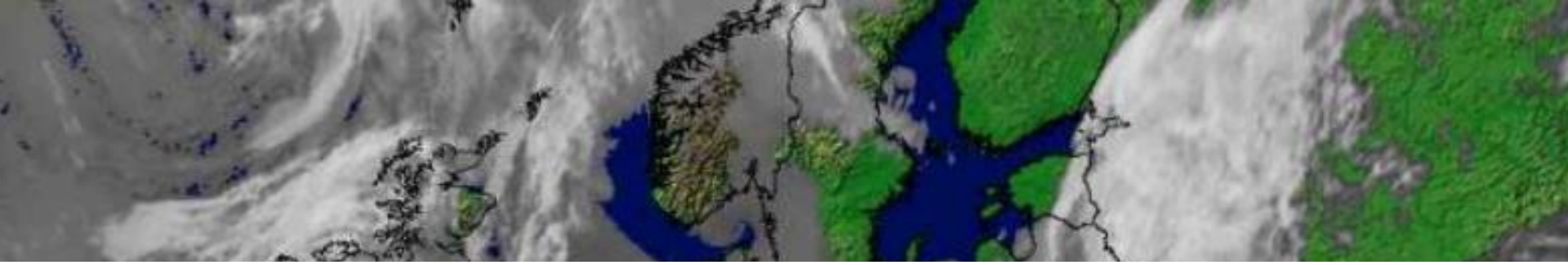




# Application of Ensemble Transform Kalman Filter in Numerical Weather Prediction

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# Numerical Weather Prediction



# Numerical Weather Prediction

- Aim: give an estimation of the future state of the atmosphere, i.e. give a weather forecast.
- How? Numerically solve the **hydro-thermodynamic equations**, which describe the atmospheric flow.
- This is a system of non-linear Partial Differential Equations.

$$\frac{d\bar{v}}{dt} = -\frac{1}{\rho} \nabla p + \bar{g} - 2\bar{\Omega} \times \bar{v}$$

$\bar{v}$ : wind

$$\frac{d\rho}{dt} = -\rho \cdot \text{div} \bar{v}$$

$\rho$ : density

$$\frac{dQ}{dt} = c_p \frac{dT}{dt} - \alpha \frac{dp}{dt}$$

$T$ : temperature

$$\frac{dq}{dt} = -\frac{1}{\rho} M$$

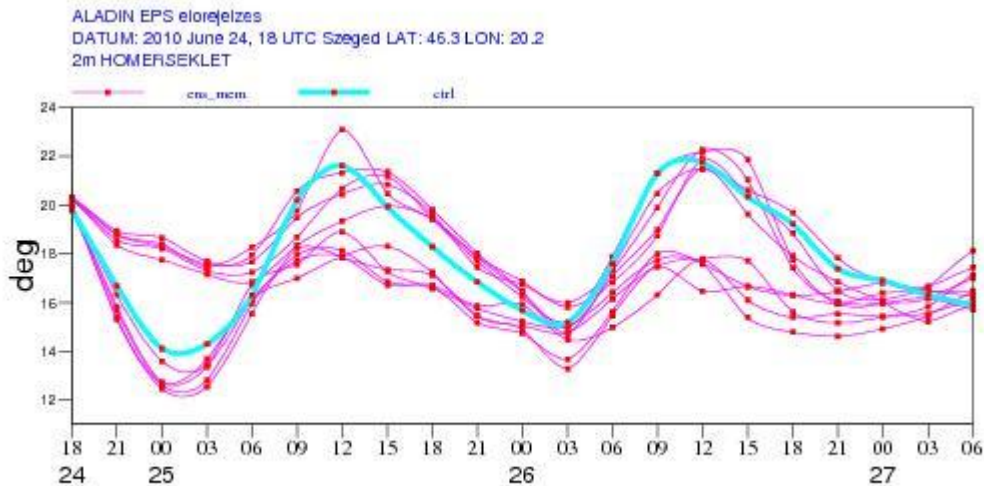
$p$ : pressure

$q$ : humidity

$$p = \rho RT$$

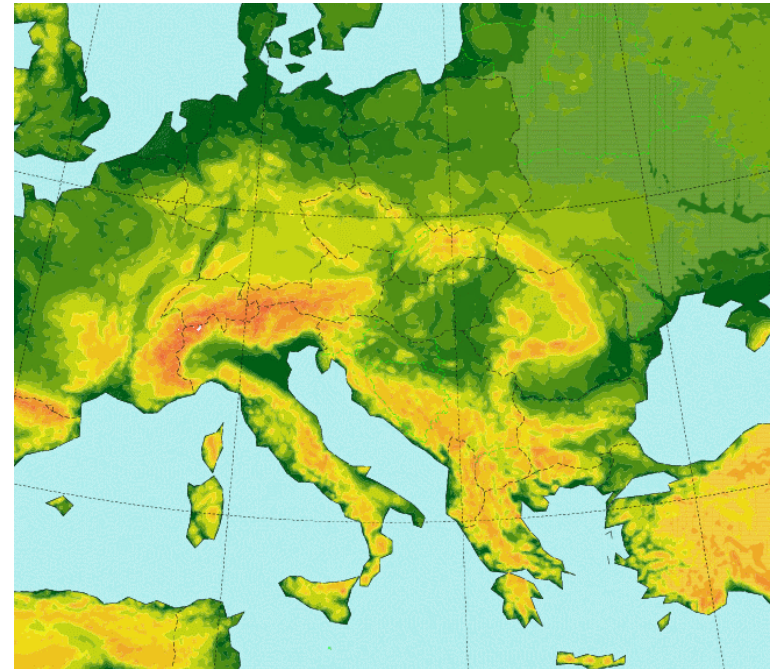
# Data Assimilation

- The model needs **initial-** and lateral boundary **conditions**.
- The models are extremely sensitive to changes in the initial condition, so their precise estimation is crucial in NWP.
- **Data Assimilation** is the method to create the best possible estimate of the atmospheric state at the initial time, i.e. the analysis.



# Operational ALADIN / HU

- ALADIN NWP model (cycle 33)
- LAM domain over Central Europe
- 8 km horizontal resolution, 49 vertical levels
- LBC from the IFS model (ECMWF)
- IC: local data assimilation
- analysis 6 hourly





# Data Assimilation

- Analysis: the best possible estimate of the state vector.
- Two kind of information are used:
  - observations and
  - background information (a forecast from a previous analysis)
- The analysis is obtained by the minimization of the cost function:

$$E|x_t - x_a|^2 \rightarrow \min$$

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$$J(x) = \frac{1}{2} \underbrace{(x_f - x)P_f^{-1}(x_f - x)^T}_{\text{distance from the background weighted by the reliability of the forecast}} + \frac{1}{2} \underbrace{(y - H(x))R^{-1}(y - H(x))^T}_{\text{distance from the observation weighted by the reliability of the observation}}$$

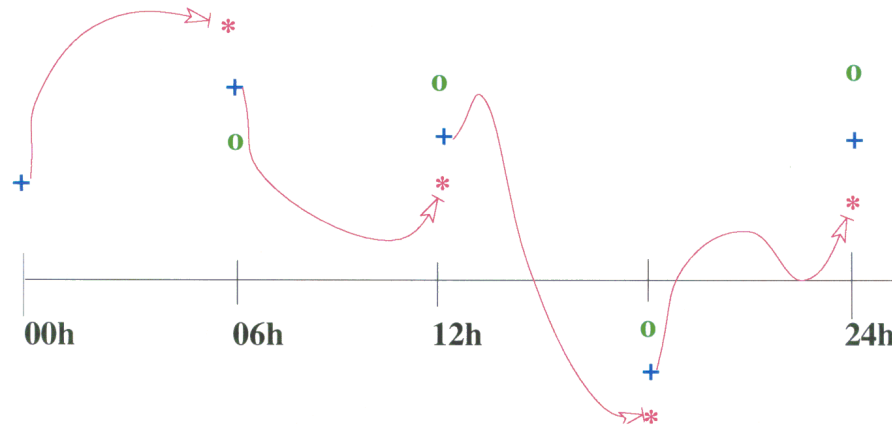
$$x_a = \arg \min J(x)$$

distance from the background  
weighted by the reliability of the forecast

distance from the observation  
weighted by the reliability of the observation

$x_t, x_a, x_f \in \mathbb{R}^n$  model space  
(true state, analysis, forecast)  
Observations:  $y \in \mathbb{R}^p$   
Observation space  $\subset \mathbb{R}^p$  ( $p \approx 10^5$ )  
Observation operator  $H: \mathbb{R}^n \rightarrow \mathbb{R}^p$   
 $P_f$ : forecast error covariance matrix  
 $R$ : observation error covariance matrix

# Data Assimilation



background + new observation  $\rightarrow$  analysis

$$x_f(k+1) = Mx_a(k)$$

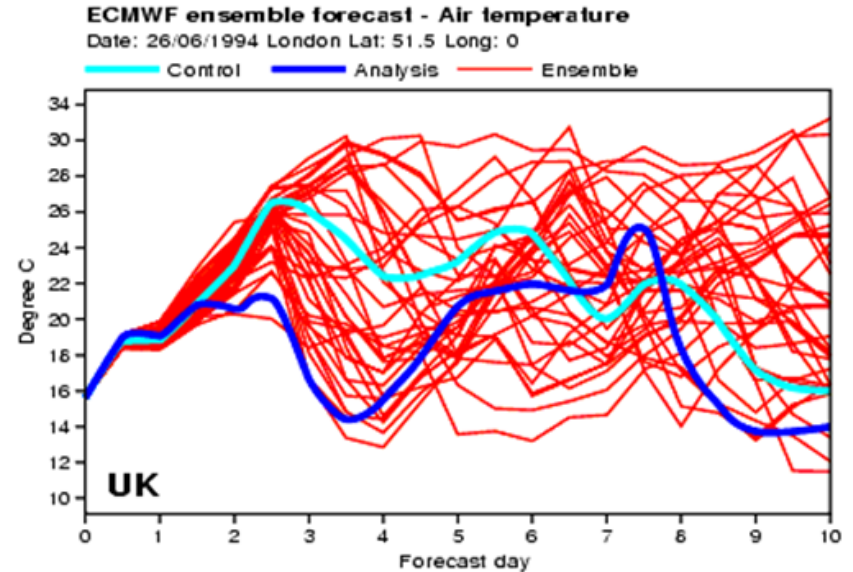
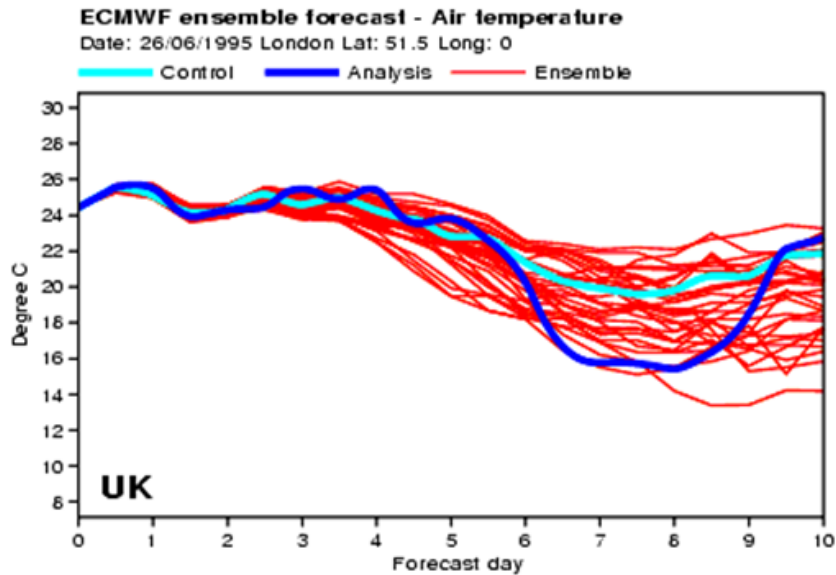
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Forecast error statistics are time-invariant in the model.



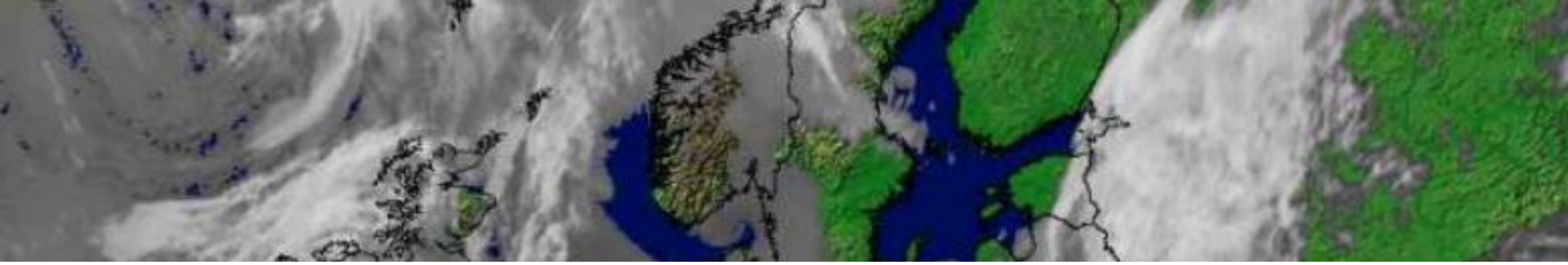
# Forecast error



Ensemble forecast  
London

1 year difference: 26/06/1995 and 26/06/1994

Aim of the research: flow-dependent computation of the forecast error covariance matrix.



# Ensemble Transform Kalman Filter



# Kalman Filter

Time evolution of forecast error:

$$P_f(k+1) = MP_a(k)M^T + Q$$

$M$  linear model

$P_a$  analysis error

$Q$  model error

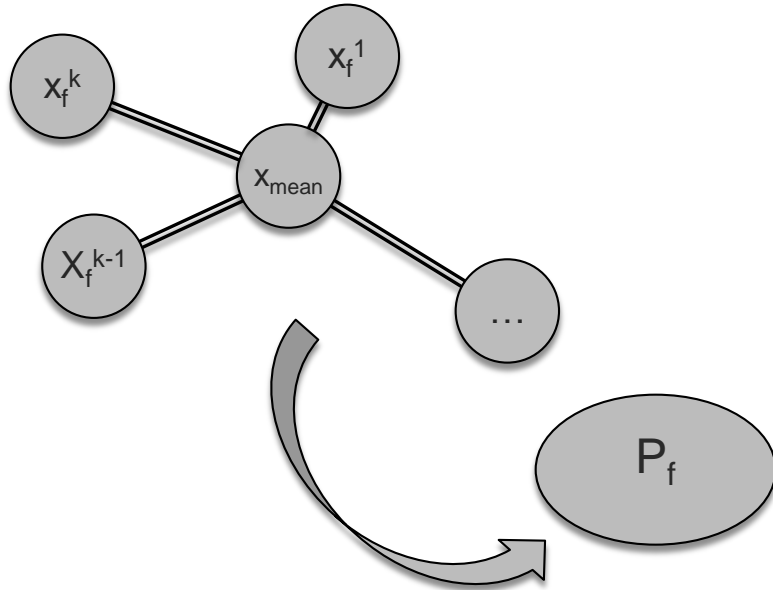


Kálmán Rudolf Emil  
(1930- )

# Ensemble Transform KF

The forecast error ( $P_f$ ) is represented by the forecast ensemble.

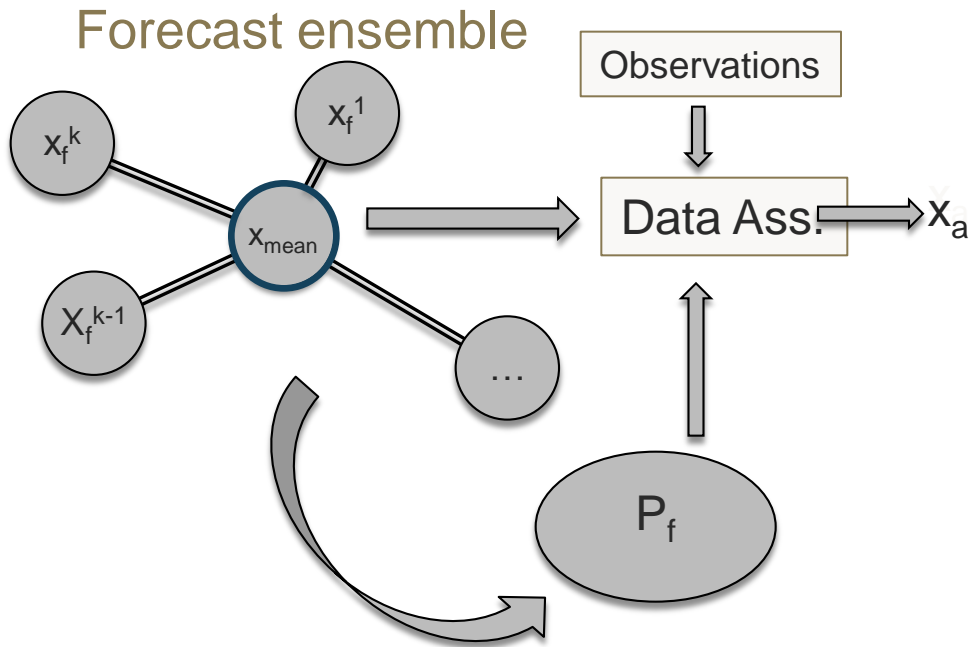
Forecast ensemble



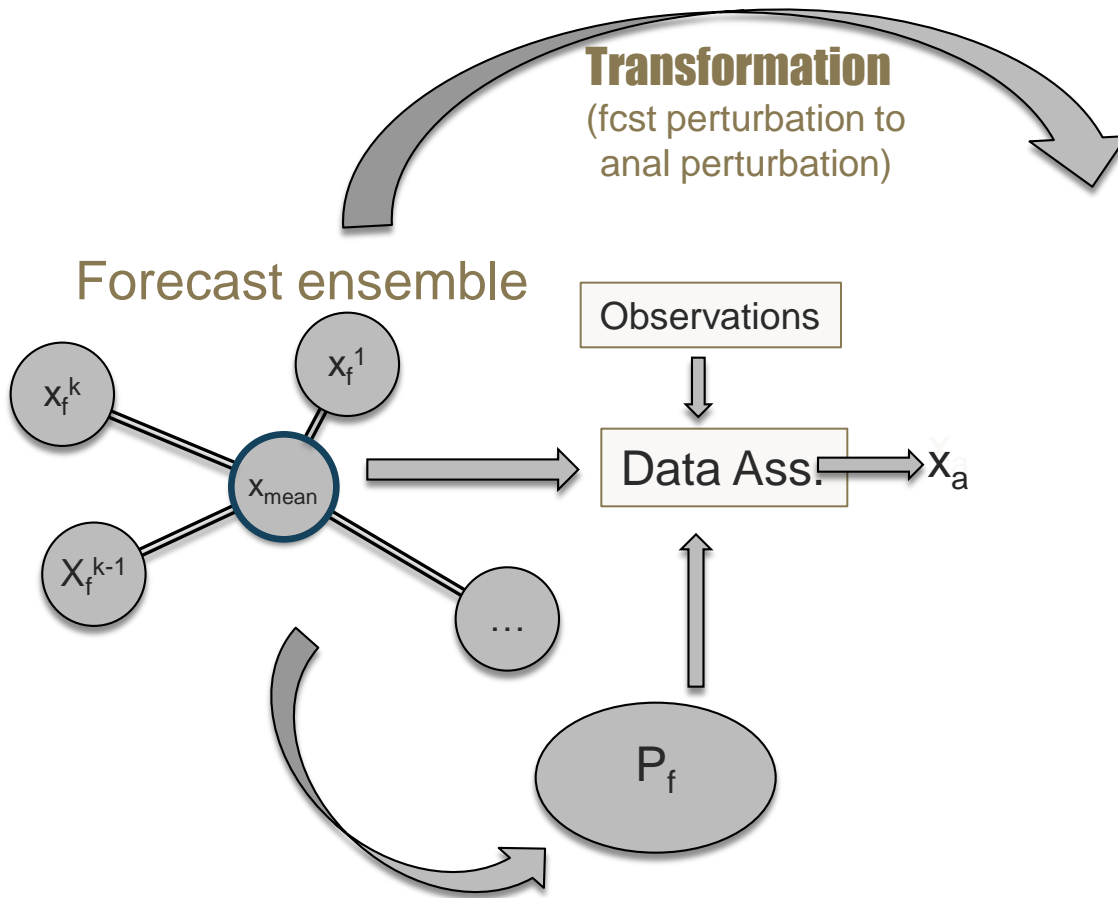


# Ensemble Transform KF

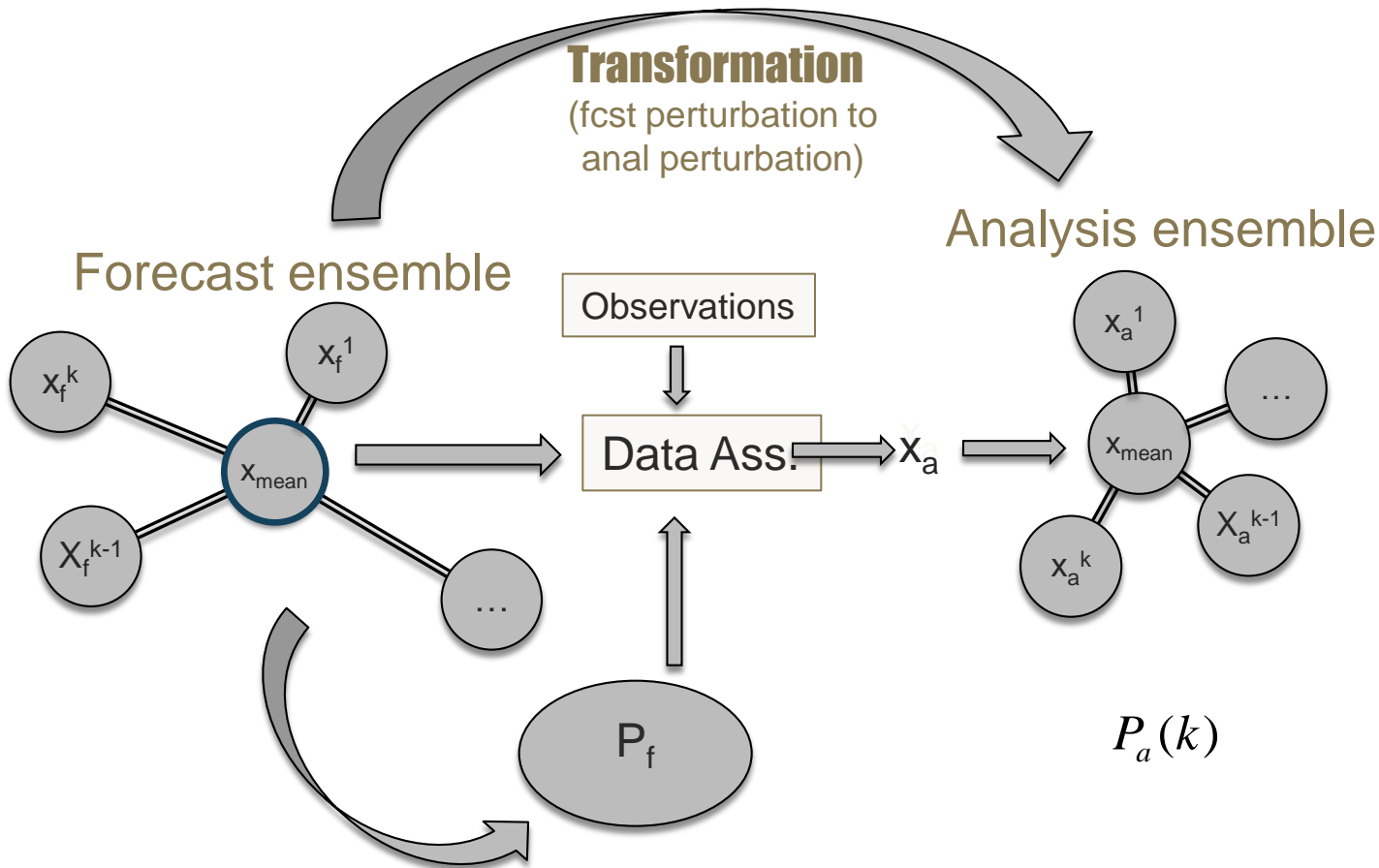
The forecast error ( $P_f$ ) is represented by the forecast ensemble.



# Ensemble Transform KF

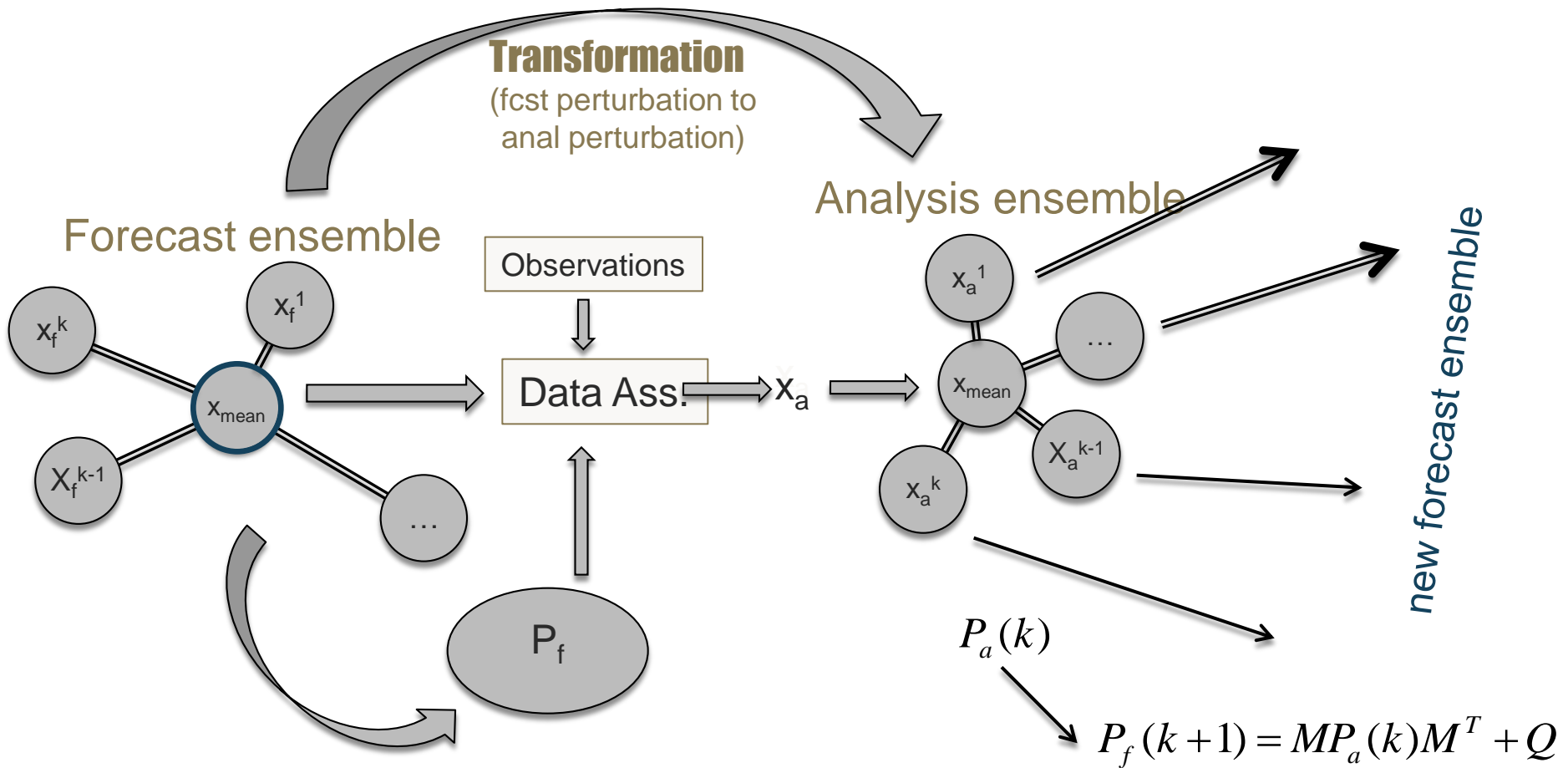


# Ensemble Transform KF



The analysis error will be represented by the analysis ensemble.

# Ensemble Transform KF



The new  $P_f$  will be represented by the new forecast ensemble.